Digital SAT Math

Geometry — Practice Set III

Hard and Very Hard Module 2 Questions

6 Questions | Full Solutions | Mistake-Pattern Analysis

Question 1 [VERY HARD]

A circle of radius 5 is inscribed in a square with side length 10. What is the area of the region inside the square but outside the circle, in terms of π?

(Student-produced response)

Shortcut: Inscribed Circle Radius = half the square's side length

Solution: Square area = $10^2 = 100$. Circle area = $\pi(5)^2 = 25\pi$. Region = $100 - 25\pi$.

Mistake: Concept gap: student uses the diameter (10) as the circle's radius in the area formula, computing 100π instead of 25π for the circle, since the circle's diameter matches the square's side.

Question 2 [VERY HARD]

A sphere has a surface area of 100π. What is its volume, in terms of π?

(Student-produced response)

Shortcut: Surface Area = $4\pi r^2$; solve for r, then apply volume = $(4/3)\pi r^3$

Solution: Surface area = $4\pi r^2 = 100\pi$, so $r^2 = 25$, giving $r = 5$. Volume = $(4/3)\pi(5)^3 = (4/3)(125)\pi = 500\pi/3$.

Mistake: Overcomplication: student tries to find the diameter first and convert between diameter and radius multiple times instead of solving directly for r^2 from the surface area formula.

Question 3 [VERY HARD]

In a 30-60-90 triangle, the longer leg measures 9. What is the length of the hypotenuse?

(Student-produced response)

Shortcut: 30-60-90 Reversed: longer leg = shorter leg × √3; shorter leg = longer/√3

Solution: The longer leg equals the shorter leg times $\sqrt{3}$, so the shorter leg is $9/\sqrt{3} = 3\sqrt{3}$. The hypotenuse is twice the shorter leg: $6\sqrt{3}$.

Mistake: Concept gap: student treats the given side as the shorter leg instead of the longer leg, applying the ratios in the wrong order and producing an incorrect hypotenuse.

Question 4 [VERY HARD]

In a right triangle, $\sin(\theta) = 5/13$ for an acute angle θ. What is $\cos(\theta)$?

(Student-produced response)

Shortcut: Pythagorean Triple: sin=5/13 means 5-12-13 triangle; cos=12/13

Solution: This corresponds to a 5-12-13 triangle. Since $\sin(\theta)$ = opposite/hypotenuse = 5/13, the adjacent side is 12, so $\cos(\theta) = 12/13$.

Mistake: Overcomplication: student applies the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ with decimal approximations instead of recognizing the 5-12-13 Pythagorean triple directly, introducing unnecessary rounding.

Question 5 **[VERY HARD]**

A sector has an arc length of 8π and a central angle of 90°. What is the radius of the circle?

(Student-produced response)

Shortcut: Arc Length = (central angle / 360°) × 2πr; solve for r

Solution: A 90° sector is 1/4 of the circle, so the arc length is 1/4 of the circumference: $8\pi = (1/4)(2\pi r) = (\pi r)/2$. Solving gives $r = 16$.

Mistake: Concept gap: student sets the full circumference equal to 8π instead of recognizing that 8π is only the arc length for a quarter of the circle, producing an incorrect radius.

Question 6 **[VERY HARD]**

A cylinder and a cone have the same radius. The cylinder has a height of 4. If the cone has the same volume as the cylinder, what is the height of the cone?

(Student-produced response)

Shortcut: Equal Volumes: set cylinder = cone formula and solve for the unknown height

Solution: Cylinder volume = $\pi r^2(4)$. Cone volume = $(1/3)\pi r^2 h$. Setting these equal: $4 = (1/3)h$, so $h = 12$.

Mistake: Concept gap: student sets the heights equal directly instead of setting the full volume expressions equal, missing the factor of 3 introduced by the cone's volume formula.