

Geometry — Practice Set II

Trigonometry and Circles: Medium to Hard

6 Questions | Full Solutions | Mistake-Pattern Analysis

Question 1 [MEDIUM]

In a right triangle, one acute angle measures 30° and the hypotenuse is 10. What is the length of the side opposite the 30° angle?

(Student-produced response)

Algebraic: In a 30-60-90 triangle, the side opposite the 30° angle is half the hypotenuse: $10 \times \sin(30^\circ) = 10 \times 0.5 = 5$.

Desmos: Graph the triangle's coordinates and measure the side directly.

Faster path: Algebraic. Recognizing this as a special right triangle (ratio 1: $\sqrt{3}$:2) is faster than setting up coordinates.

Mistake: **Concept gap**: student uses cosine instead of sine for the side opposite the given angle, computing $10 \times \cos(30^\circ) \approx 8.66$ instead.

Question 2 [MEDIUM]

A 45-45-90 triangle has a hypotenuse of $8\sqrt{2}$. What is the length of each leg?

(Student-produced response)

Algebraic: In a 45-45-90 triangle, the hypotenuse equals $\text{leg} \times \sqrt{2}$. So $\text{leg} = 8\sqrt{2} / \sqrt{2} = 8$.

Desmos: Not useful, this is a direct special-triangle ratio.

Faster path: Algebraic, recognizing the 45-45-90 ratio directly avoids any trig evaluation at all.

Mistake: **Concept gap**: student divides $8\sqrt{2}$ by 2 instead of by $\sqrt{2}$, computing $4\sqrt{2}$ instead of 8.

Question 3 [HARD]

A right triangle has a hypotenuse of 13 and one leg of length 5. What is the length of the other leg?

(Student-produced response)

Algebraic: By the Pythagorean theorem, the other leg $= \sqrt{(13^2 - 5^2)} = \sqrt{(169 - 25)} = \sqrt{144} = 12$.

Desmos: Not useful, this is a direct formula substitution.

Faster path: Algebraic, always. Recognizing 5-12-13 as a known Pythagorean triple speeds this up further.

Mistake: **Concept gap**: student adds 13^2 and 5^2 instead of subtracting, computing $\sqrt{194}$ instead of $\sqrt{144}$.

Question 4 [MEDIUM]

A circle has a radius of 6. What is its circumference, in terms of π ?

(Student-produced response)

Algebraic: Circumference $= 2\pi r = 2\pi(6) = 12\pi$.

Desmos: Not useful, this is a direct formula substitution.

Faster path: Algebraic, always.

Mistake: **Concept gap**: student uses the area formula instead of the circumference formula, computing 36π instead of 12π .

Question 5 [MEDIUM]

A circle has a diameter of 14. What is its area, in terms of π ?

(Student-produced response)

Algebraic: Radius = $14/2 = 7$. Area = $\pi r^2 = \pi(7)^2 = 49\pi$.

Desmos: Not useful, this is a direct formula substitution.

Faster path: Algebraic, always.

Mistake: **Concept gap**: student uses the diameter directly in the area formula instead of converting to the radius first, computing $\pi(14)^2 = 196\pi$.

Question 6 [HARD]

An inscribed angle in a circle intercepts an arc measuring 80° . What is the measure of the inscribed angle?

(Student-produced response)

Algebraic: An inscribed angle is always half the measure of its intercepted arc: $80/2 = 40^\circ$.

Desmos: Not useful, this is a direct application of the inscribed angle theorem.

Faster path: Algebraic, recognizing the theorem directly.

Mistake: **Concept gap**: student reports the arc measure itself (80°) as the inscribed angle, confusing it with the central angle, which does equal the arc measure.