

Problem-Solving and Data Analysis — Free Sample

6 Questions · Easy to Very Hard · Full Solutions · Mistake-Pattern Analysis

Question 1 [EASY]

A recipe calls for 3 cups of water for every 5 cups of rice. For 15 cups of rice, how many cups of water are needed?

(Student-produced response)

Algebraic: Set up the proportion $\frac{3}{5} = \frac{x}{15}$. Since 15 is 3 times 5, $x = 3$ times 3 = 9.

Desmos: Not useful. Direct scaling of the ratio.

Faster path: Algebraic, always, for a direct proportional scale-up.

Mistake: **Concept gap**: student inverts the ratio and computes $\frac{5}{3}$ times 15 = 25 instead of $\frac{3}{5}$ times 15 = 9.

Question 2 [EASY]

What is the median of the data set 4, 7, 7, 9, 12, 15?

(Student-produced response)

Algebraic: With 6 sorted values, the median is the average of the 3rd and 4th values: $(7 + 9)$ divided by 2 = 8.

Desmos: Enter the data into a Desmos table and use the median() function.

Faster path: Algebraic. Averaging two middle values is faster than setting up a table for six numbers.

Mistake: **Concept gap**: student reports the 4th value (9) as the median instead of averaging the 3rd and 4th values for an even-sized data set.

Question 3 [MEDIUM]

A line of best fit is $y = 0.5x + 3$. A data point has an actual y-value of 10 when $x = 8$. What is the residual for this point?

(Student-produced response)

Algebraic: Predicted value: $y = 0.5(8) + 3 = 7$. Residual = actual minus predicted = 10 minus 7 = 3.

Desmos: Graph the line and plot the point. The residual is the vertical distance above the line.

Faster path: Algebraic. Substituting and subtracting is faster than graphing for a single point.

Mistake: **Concept gap**: student computes predicted minus actual instead of actual minus predicted, getting the sign of the residual backward.

Question 4 [MEDIUM]

A survey of 200 students shows 40 ninth-graders like math and 60 do not, while 70 tenth-graders like math and 30 do not. What is the probability that a randomly selected student likes math?

(Student-produced response)

Algebraic: Total who like math: $40 + 70 = 110$. Probability = 110 divided by 200 = 0.55.

Desmos: Not useful. Direct table read.

Faster path: Algebraic, always, for a single-proportion table read.

Mistake: **Concept gap**: student uses only one grade group, dividing 70 by 100 or 40 by 100, instead of combining both groups before dividing by the full total.

Question 5 [HARD]

Group A has 10 students with a mean score of 70. Group B has 15 students with a mean score of 80. What is the combined mean score of all 25 students?

(Student-produced response)

Algebraic: Total points = (10 times 70) + (15 times 80) = 700 + 1200 = 1900. Combined mean = 1900 divided by 25 = 76.

Desmos: Not useful. Weighted mean requires arithmetic, not graphing.

Faster path: Algebraic, always. Compute the total sum for each group, add them, then divide by the combined total.

Mistake: **Concept gap**: student averages the two group means directly: $(70 + 80) \div 2 = 75$, which is only correct when both groups are the same size.

Question 6 [VERY HARD]

A price increases by 25 percent and then decreases by 20 percent. What is the net percent change from the original price?

(Student-produced response)

Shortcut: Multiply remaining-value factors: $1.25 \times 0.80 = 1.00$. Net change = 0 percent.

Solution: The combined factor is $1.25 \times 0.80 = 1.00$, meaning the final price equals the original. Net change = 0 percent.

Mistake: **Concept gap**: student adds 25 percent and minus 20 percent to get a net 5 percent increase, applying an additive rule to a multiplicative process.