

Advanced Math — Free Sample

6 Questions · Easy to Very Hard · Full Solutions · Mistake-Pattern Analysis

Question 1 [EASY]

Which expression is equivalent to $3x(x + 4)$ minus $2(x + 4)$?

- A) $(3x \text{ minus } 2)(x + 4)$
- B) $(3x + 2)(x + 4)$
- C) $3x \text{ squared} + 10x \text{ minus } 8$
- D) $3x \text{ squared minus } 10x + 8$

Algebraic: Factor out $(x + 4)$: $(x + 4)(3x \text{ minus } 2)$.

Desmos: Expand both and compare. Desmos can verify but factoring is instant.

Faster path: Algebraic. Recognising the common factor $(x + 4)$ in both terms gives the answer in one step.

Mistake: **Overcomplication**: student expands all terms first, getting $3x \text{ squared} + 12x \text{ minus } 2x \text{ minus } 8 = 3x \text{ squared} + 10x \text{ minus } 8$, and then cannot match it to the factored form in the answer choices.

Question 2 [EASY]

Which expression is equivalent to $(x + 3)$ squared minus $(x \text{ minus } 3)$ squared?

- A) $12x$
- B) $2x \text{ squared} + 18$
- C) 12
- D) 36

Algebraic: Difference of squares: $(a \text{ squared minus } b \text{ squared}) = (a+b)(a \text{ minus } b)$. Here $a = x+3$, $b = x \text{ minus } 3$.
Product = $(2x)(6) = 12x$.

Desmos: Expand both squares and subtract. Desmos can confirm.

Faster path: Algebraic. Recognising the difference-of-squares pattern gives the answer in five seconds.

Mistake: **Concept gap**: student expands both squares fully and subtracts term by term instead of applying the difference-of-squares identity directly.

Question 3 [MEDIUM]

How many solutions does the system $y = x \text{ squared minus } 2x \text{ minus } 3$ and $y = 2x + 1$ have?

- A) Zero
- B) One
- C) Two
- D) Infinitely many

Algebraic: Substitute: $x \text{ squared minus } 2x \text{ minus } 3 = 2x + 1$. Rearrange: $x \text{ squared minus } 4x \text{ minus } 4 = 0$.
Discriminant: $16 + 16 = 32$. Positive discriminant means two solutions.

Desmos: Graph both and count intersections.

Faster path: Algebraic with discriminant check. Computing only the sign of $b \text{ squared minus } 4ac$ is faster than graphing and counting.

Mistake: **Pacing error**: student sets up the combined equation correctly but solves fully for both roots before realising the question only asked for the count.

Question 4 [MEDIUM]

A parabola has vertex (3, minus 4) and passes through the point (5, 4). What is the value of a in $f(x) = a(x \text{ minus } 3) \text{ squared minus } 4$?

(Student-produced response)

Algebraic: Substitute (5, 4): $4 = a(5 \text{ minus } 3) \text{ squared minus } 4$. $4 = 4a \text{ minus } 4$. $8 = 4a$. $a = 2$.

Desmos: Not faster. Direct substitution of the given point solves for a in two lines.

Faster path: Algebraic. The vertex form is already given, so substituting the second point is a direct one-unknown solve.

Mistake: **Concept gap**: student substitutes the vertex coordinates instead of the non-vertex point, producing $0 = 0$ and concluding a cannot be found.

Question 5 [HARD]

What is the sum of the solutions to $2x \text{ squared minus } 7x + 3 = 0$?

(Student-produced response)

Shortcut: Vieta's Formulas: sum of roots = negative b over a = negative (minus 7) over 2 = 3.5.

Solution: Using Vieta's formulas, sum = minus b over a = minus(minus 7) over 2 = $7/2 = 3.5$.

Mistake: **Overcomplication**: student applies the full quadratic formula, finds both roots separately, and then adds them, spending two minutes on a five-second calculation.

Question 6 [VERY HARD]

For what positive value of k does the equation $x \text{ squared} + kx + 25 = 0$ have exactly one real solution?

(Student-produced response)

Shortcut: Discriminant = 0: set $b \text{ squared minus } 4ac = 0$. $k \text{ squared minus } 100 = 0$. $k = 10$.

Solution: For exactly one real solution, the discriminant equals zero: $k \text{ squared minus } 4(1)(25) = 0$. $k \text{ squared} = 100$. $k = 10$ (taking the positive value).

Mistake: **Concept gap**: student tries to factor $x \text{ squared} + kx + 25$ as $(x + 5) \text{ squared} = x \text{ squared} + 10x + 25$ and gets $k = 10$ by pattern-matching, but cannot explain why or generalise the method.