

Algebra — Practice Set III

Hard and Very Hard Module 2 Questions

6 Questions | Full Solutions | Mistake-Pattern Analysis

Question 1 [VERY HARD]

For what value of k does the system $kx + 3y = 6$ and $2x + 6y = 10$ have no solution?

(Student-produced response)

Shortcut: Ratio Comparison (3 ratios, identify parallel lines)

Solution: For the lines to be parallel, the coefficient ratios must match: $k/2 = 3/6$, so $k = 1$. Checking the constants, $6/10 \neq 3/6$, confirming the lines are parallel and distinct, so $k = 1$ produces no solution.

Mistake: **Concept gap**: student sets the ratio up using only the y -coefficients and constants, ignoring the x -coefficient relationship, and solves for a different (incorrect) value.

Question 2 [VERY HARD]

For what value of m does the system $mx - 4y = 8$ and $2x - 8y = 16$ have infinitely many solutions?

(Student-produced response)

Shortcut: Ratio Comparison (all 3 ratios must match for infinite solutions)

Solution: For the lines to be identical, all ratios must match: $m/2 = -4/-8 = 0.5$, so $m = 1$. Checking constants, $8/16 = 0.5$ as well, confirming infinitely many solutions at $m = 1$.

Mistake: **Overcomplication**: student attempts to solve the system directly for x and y first, then works backward to find m , instead of comparing ratios directly.

Question 3 [VERY HARD]

If $-10 \leq -2x + 4 \leq 2$, which of the following describes all possible values of x ?

- A) $1 \leq x \leq 7$
- B) $-7 \leq x \leq -1$
- C) $-1 \leq x \leq 7$
- D) $1 \leq x \leq 14$

Shortcut: Compound Inequality: subtract, then divide by negative and double-flip

Solution: Subtract 4 from all parts: $-14 \leq -2x \leq -2$. Divide by -2 and reverse the inequality direction along with flipping it: $1 \leq x \leq 7$.

Mistake: **Concept gap**: student divides by -2 and flips the inequality signs but forgets to also reverse the order of the boundary values, writing $7 \leq x \leq 1$, which is meaningless, and then guesses based on the numbers alone.

Question 4 [VERY HARD]

A linear function f has $f(0) = b$ and a slope equal to $3b$. If $f(2) = 21$, what is the value of b ?

- A) 1
- B) 2
- C) 3
- D) 7

Shortcut: Express both slope and intercept in terms of b , solve $f(2)=21$

Solution: $f(2) = 3b(2) + b = 7b$. Setting $7b = 21$ gives $b = 3$.

Mistake: **Concept gap**: student treats the slope and intercept as two separate unknowns to solve for individually instead of recognizing both are expressed in terms of the same variable b .

Question 5 [VERY HARD]

Solution A is 100 mL at 10% concentration. How many mL of solution B at 40% concentration must be added to produce a mixture at 25% concentration?

(Student-produced response)

Shortcut: Mixture Equation: set up $0.10(100)+0.40x = 0.25(100+x)$

Solution: $0.10(100) + 0.40x = 0.25(100 + x)$, so $10 + 0.4x = 25 + 0.25x$, giving $0.15x = 15$, so $x = 100$.

Mistake: **Concept gap**: student sets up the right side as $0.25(100)$ only, forgetting that the total volume after mixing includes the added x mL as well.

Question 6 [VERY HARD]

If $3(x - 2) \geq 2x + 1$ and $x < 10$, what is the smallest integer value of x that satisfies both conditions?

- A) 6
- B) 7
- C) 8
- D) 9

Shortcut: Solve inequality, then identify smallest integer satisfying both conditions

Solution: Distribute: $3x - 6 \geq 2x + 1$, so $x \geq 7$. Combined with $x < 10$, the smallest integer satisfying both is 7.

Mistake: **Concept gap**: student solves only the first inequality and reports the boundary value without checking it also satisfies the second condition, which matters more when the two ranges do not fully overlap.