

Advanced Math — Practice Set III

Hard and Very Hard Module 2 Questions

6 Questions | Full Solutions | Mistake-Pattern Analysis

Question 1 [VERY HARD]

For what positive value of k does $x^2 + kx + 25 = 0$ have exactly one real solution?

(Student-produced response)

Shortcut: Discriminant = 0: set $b^2 - 4ac = 0$ and solve for k

Solution: For exactly one solution, the discriminant must be zero: $k^2 - 100 = 0$, so $k = \pm 10$. The positive value is 10.

Mistake: **Concept gap**: student sets up the discriminant correctly but reports both ± 10 instead of selecting only the positive value the question asked for.

Question 2 [VERY HARD]

A bacteria population triples every 5 hours, starting at 60 bacteria. What is the population after 10 hours?

(Student-produced response)

Shortcut: Exponential Model: exponent = t/period , not $t \times \text{period}$

Solution: $P(t) = 60(3)^{t/5}$. At $t = 10$, the exponent is 2, so $P(10) = 60(3^2) = 60(9) = 540$.

Mistake: **Concept gap**: student builds the exponent as $5t$ or $t \times 5$ instead of $t/5$, inverting the relationship between the tripling period and the exponent.

Question 3 [VERY HARD]

A parabola $y = x^2 - 4x + c$ has its vertex on the x -axis. What is the value of c ?

(Student-produced response)

Shortcut: Discriminant = 0: vertex on x -axis means one real solution

Solution: A vertex on the x -axis means the discriminant is zero: $16 - 4c = 0$, so $c = 4$.

Mistake: **Concept gap**: student finds the vertex x -coordinate (2) correctly but then sets c equal to that x -coordinate instead of substituting it back to find the y -value condition.

Question 4 [VERY HARD]

A line $y = 3x + b$ is tangent to the parabola $y = x^2 + 2x + 5$. What is the value of b ?

(Student-produced response)

Shortcut: Tangency Condition: substitute line into parabola, set discriminant to 0

Solution: Substitute: $x^2 + 2x + 5 = 3x + b$, so $x^2 - x + (5-b) = 0$. For tangency, the discriminant is zero: $1 - 4(5-b) = 0$, giving $4b = 19$, so $b = 4.75$.

Mistake: **Pacing error**: student correctly sets up the discriminant equation but re-expands the $(5-b)$ term a second time to "double check" before solving, adding time to an already-correct setup.

Question 5 [VERY HARD]

A quadratic equation $x^2 - px + 36 = 0$ has two positive integer roots. If one root is 4, what is the value of p ?

(Student-produced response)

Shortcut: Vieta's Formulas: product of roots = c/a ; sum of roots = $-b/a$

Solution: By Vieta's formulas, the product of the roots is 36, so the other root is $36/4 = 9$. The sum of the roots is $4 + 9 = 13$, and since the sum equals p (note the equation's $-p$), $p = 13$.

Mistake: **Overcomplication**: student substitutes $x = 4$ into the equation to solve for p directly through a more complicated rearrangement instead of using the product-of-roots shortcut to find the second root first.

Question 6 [VERY HARD]

A quadratic function $f(x) = ax^2 + bx + c$ satisfies $f(0) = 5$, $f(1) = 8$, and $f(-1) = 6$. What is the value of $f(2)$?

(Student-produced response)

Shortcut: System of Three Conditions: use $f(0)$, $f(1)$, $f(-1)$ to solve for a , b , c

Solution: $f(0) = c = 5$. $f(1) = a + b + c = 8$, so $a + b = 3$. $f(-1) = a - b + c = 6$, so $a - b = 1$. Adding these two equations: $2a = 4$, so $a = 2$ and $b = 1$. Then $f(x) = 2x^2 + x + 5$, and $f(2) = 8 + 2 + 5 = 15$.

Mistake: **Overcomplication**: student tries to guess values of a , b , and c by inspection instead of setting up and solving the small system of three equations directly, often getting stuck or introducing an arithmetic error along the way.