

Advanced Math — Practice Set II

Quadratic Functions and Nonlinear Systems: Medium to Hard

6 Questions | Full Solutions | Mistake-Pattern Analysis

Question 1 [MEDIUM]

The function $f(x) = (x - 3)^2 + 5$ is graphed in the xy -plane. What is the vertex of the graph?

- A) (3, 5)
- B) (-3, 5)
- C) (3, -5)
- D) (5, 3)

Algebraic: In vertex form $a(x-h)^2 + k$, the vertex is (h, k) . Here $h = 3$ and $k = 5$, so the vertex is $(3, 5)$.

Desmos: Graph the function directly and read the vertex coordinates from the lowest point of the curve.

Faster path: Algebraic. The vertex form already displays the answer; graphing to confirm something already given in the question wastes time.

Mistake: **Concept gap**: student flips the sign on h , reading the vertex as $(-3, 5)$ since the form shows " $x - 3$ " with a minus sign.

Question 2 [MEDIUM]

The graph of $y = x^2$ is transformed to $y = (x - 4)^2 + 2$. Which statement correctly describes the transformation?

- A) Shift left 4, up 2
- B) Shift right 4, up 2
- C) Shift right 4, down 2
- D) Shift left 4, down 2

Algebraic: Inside the parentheses, $(x - 4)$ shifts the graph right by 4, since the transformation is the opposite of the sign shown. Adding 2 outside shifts the graph up by 2.

Desmos: Graph both $y = x^2$ and $y = (x-4)^2 + 2$ on the same screen and visually compare the shift.

Faster path: Algebraic, recognizing the transformation rule directly is faster than graphing two functions and estimating the shift visually.

Mistake: **Concept gap**: student reads " $x - 4$ " as a shift left, forgetting that horizontal shifts inside the parentheses move opposite to the sign shown.

Question 3 [HARD]

An exponential function is given by $f(x) = 500(1.08)^x$, modeling an investment's value in dollars after x years. What is the annual growth rate, as a percent?

- A) 1.08%
- B) 8%
- C) 80%
- D) 108%

Algebraic: In the form $a(1+r)^x$, the growth rate is r . Here $1 + r = 1.08$, so $r = 0.08$, or 8%.

Desmos: Graph the function and compare values at $x = 0$ and $x = 1$; the percent increase between them is the growth rate.

Faster path: Algebraic, reading r directly from the base of the exponential is faster than computing and comparing two function values.

Mistake: **Concept gap**: student reports the full base (1.08, or 108%) as the growth rate instead of just the part above 1.

Question 4 [MEDIUM]

What is the sum of the x -coordinates of the two solutions to the system $y = x^2$ and $y = x + 2$?

(Student-produced response)

Algebraic: Substitute: $x^2 = x + 2$, so $x^2 - x - 2 = 0$. By Vieta's formulas, the sum of the roots is $-b/a = 1$.

Desmos: Graph both equations and add the x -coordinates of the two intersection points shown.

Faster path: Algebraic, Vieta's formula gives the sum without needing to find either x -value individually.

Mistake: **Overcomplication**: student factors the combined equation completely to find both x -values (2 and -1) and adds them by hand instead of reading the sum directly from the coefficients.

Question 5 [MEDIUM]

How many solutions does the system $y = x^2 - 4$ and $y = 2x - 4$ have?

- A) Zero
- B) One
- C) Two
- D) Infinitely many

Algebraic: Substitute: $x^2 - 4 = 2x - 4$, so $x^2 - 2x = 0$, giving $x(x-2) = 0$, so $x = 0$ or $x = 2$. Two distinct solutions.

Desmos: Graph both equations and count the intersection points.

Faster path: Desmos, slightly, since counting visible intersections is about as fast as solving and avoids an algebra slip.

Mistake: **Concept gap**: student finds the combined equation but stops after factoring out x , treating $x = 0$ as the only solution and forgetting the second factor.

Question 6 [HARD]

What is the positive solution for x in the system $y = x^2 - 2x$ and $y = 3$?

(Student-produced response)

Algebraic: Substitute: $x^2 - 2x = 3$, so $x^2 - 2x - 3 = 0$, giving $(x-3)(x+1) = 0$, so $x = 3$ or $x = -1$. The positive solution is 3.

Desmos: Graph $y = x^2 - 2x$ and $y = 3$, and read the positive x -coordinate of the intersection.

Faster path: Algebraic, factoring is quick here and avoids reading graph coordinates to a precise value.

Mistake: **Wording trap**: student finds both solutions correctly but reports the negative one, -1 , instead of the positive value the question asked for.