Integration by Substitution

Recognition Training · Set I

Course	IB Mathematics: Analysis & Approaches HL
Topic	Topic 5 — Calculus
Level	Easy → Medium
Questions	6
Total marks	28
Instructions	Show all working. **M1** = method mark. **A1** = accuracy mark. **R1** = reasoning mark. Do not use a calculator unless stated.

BEFORE YOU BEGIN

Integration by substitution succeeds or fails in the first three seconds. Before writing any working, identify: (1) the inner function u, and (2) whether its derivative — or a scalar multiple — also appears in the integrand. If it does not, substitution is the wrong method. Confirm the structure *before* you begin.

Question 1 Easy [4 marks]

Find the following integral.

$$\int 2x\,(x^2+3)^4\,dx$$

WARNING — RECOGNITION TRAP

The factor $2x$ is the exact derivative of x^2+3. Students often expand $(x^2+3)^4$ instead — a six-line calculation with high error probability. Recognise the composite structure and substitute immediately.

Question 2 Easy [4 marks]

Find the following integral.

$$\int \cos(3x+1)\,dx$$

WARNING — RECOGNITION TRAP

The inner function is $(3x+1)$, derivative 3. Students forget to divide by the derivative of the inner function and write $\sin(3x+1)+C$ instead of $\frac{1}{3}\sin(3x+1)+C$. The missing factor of $\frac{1}{3}$ costs both method and accuracy marks.

Question 3 Easy–Medium [5 marks]

Find the following integral.

$$\int x\,e^{x^2}\,dx$$

WARNING — RECOGNITION TRAP

The inner function is x^2, derivative $2x$. The factor x is present but not $2x$. Introduce a compensating factor of $\frac{1}{2}$, then substitute. Do not attempt integration by parts here.

Question 4 Medium [5 marks]

Find the following integral.

$$\int \frac{2x+1}{x^2+x-3}\,dx$$

WARNING — RECOGNITION TRAP

Check whether the numerator equals the derivative of the denominator before attempting any other method. Here $\frac{d}{dx}(x^2+x-3) = 2x+1 =$ numerator exactly. This gives $\ln|\text{denominator}|+C$ directly. Students who miss this attempt partial fractions unnecessarily.

Question 5 Medium [5 marks]

Difficulty: Medium

Evaluate the following definite integral, showing clearly how the limits change under substitution.

$$\int_0^2 3x^2\,(x^3+1)^2\,dx$$

WARNING — RECOGNITION TRAP

When $u = x^3 + 1$, the limits $x = 0$ and $x = 2$ must be converted: $u(0) = 1$, $u(2) = 9$. Students apply the substitution to the integrand correctly but evaluate using the original x-limits. Show the limit change on a separate line — this is explicitly tested in IB AA HL.

Question 6 Easy–Medium [5 marks]

Find the following integral.

$$\int \sin^3(x)\,\cos(x)\,dx$$

WARNING — RECOGNITION TRAP

Let $u = \sin(x)$, then $du = \cos(x)\,dx$. The integral becomes $\int u^3\,du$ directly. Students who miss this apply the identity $\sin^2(x) = \frac{1-\cos 2x}{2}$, introducing double-angle expressions unnecessarily. Check for the $f(g(x)) \cdot g'(x)$ pattern first — this is a two-line solution.

M1 = method mark (correct process). **A1** = accuracy mark (correct result). **R1** = reasoning mark.

Solution — Question 1

Let	$u = x^2 + 3 \Rightarrow \dfrac{du}{dx} = 2x \Rightarrow 2x\,dx = du$	**M1**
Integral	$\int u^4\,du$	**M1**
Integrate	$\dfrac{u^5}{5} + C$	**A1**
Substitute back	$\dfrac{(x^2+3)^5}{5} + C$	**A1**

Final answer: $\dfrac{(x^2+3)^5}{5} + C$

Solution — Question 2

Let	$u = 3x + 1 \Rightarrow du = 3\,dx \Rightarrow dx = \dfrac{1}{3}\,du$	**M1**
Integral	$\dfrac{1}{3}\int \cos u\,du$	**M1**
Integrate	$\dfrac{1}{3}\sin u + C$	**A1**
Substitute back	$\dfrac{1}{3}\sin(3x+1) + C$	**A1**

Final answer: $\dfrac{1}{3}\sin(3x+1) + C$

Solution — Question 3

Let	$u = x^2 \Rightarrow du = 2x\,dx \Rightarrow x\,dx = \dfrac{1}{2}\,du$	**M1**
Integral	$\dfrac{1}{2}\int e^u\,du$	**M1**
Integrate	$\dfrac{1}{2}e^u + C$	**A1**
Substitute back	$\dfrac{1}{2}e^{x^2} + C$	**A1**

Final answer: $\dfrac{1}{2}e^{x^2} + C$

Solution — Question 4

Recognise	$\dfrac{d}{dx}(x^2+x-3) = 2x+1 \;=\; \text{numerator}$	**R1**
Pattern	$\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert + C$	**M1**
Therefore	$\ln\lvert x^2+x-3\rvert + C$	**A1**

Final answer: $\ln\lvert x^2+x-3\rvert + C$

Solution — Question 5

Let	$u = x^3+1 \;\Rightarrow\; du = 3x^2\,dx$	**M1**
Change limits	$x=0 \Rightarrow u=1 \qquad x=2 \Rightarrow u=9$	**M1**
Integral	$\displaystyle\int_1^9 u^2\,du$	**M1**
Integrate	$\left[\dfrac{u^3}{3}\right]_1^9 = \dfrac{9^3}{3} - \dfrac{1^3}{3} = \dfrac{729-1}{3}$	**A1**
Evaluate	$\dfrac{728}{3} \approx 242.7$	**A1**

Final answer: $\dfrac{728}{3}$

Solution — Question 6

Recognise	$f(g(x))\cdot g'(x)$ form: $\;f(u)=u^3,\;\; g(x)=\sin x$	**M1**
Let	$u = \sin x \;\Rightarrow\; du = \cos x\,dx$	**M1**
Integral	$\displaystyle\int u^3\,du$	**M1**
Integrate	$\dfrac{u^4}{4} + C$	**A1**
Substitute back	$\dfrac{\sin^4 x}{4} + C$	**A1**

Final answer: $\dfrac{\sin^4 x}{4} + C$
