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Trigonometry

Mistake Analysis – Set III

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	38 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Sinusoidal graphs: $f(x) = a\sin(bx) + d$ has amplitude $|a|$, period $\dfrac{360}{b}$ (or $\dfrac{2\pi}{b}$), and midline $y = d$.

Solving with double angles: identify which form of $\cos 2x$ converts the equation to a single variable.

Bearing problems: sketch the scenario. The angle at the turning vertex is found by comparing the two bearings.

Identity proofs: work on one side only (usually the more complicated side). Never cross-multiply or move terms between sides.

Question 1

Hard – Paper 2 [6 marks]

Sketch the graph of $f(x) = 2\sin(3x) + 1$ for $0 \le x \le 120$, clearly labelling the amplitude, period, and range.

MISTAKE ANALYSIS

Amplitude $= |2| = 2$. *Period* $= \dfrac{360}{3} = 120$. *Midline* $y = 1$. *Range:* $y \in [-1, 3]$. *Maximum* 3 *at* $x = 30$; *minimum* -1 *at* $x = 90$; *passes through* $(0, 1)$ *and* $(60, 1)$ *and* $(120, 1)$. *Students who state the period as* $3 \times 360 = 1080$ *multiply instead of dividing. The coefficient inside the function compresses the period: period* $= 360/b$, *so* $b = 3$ *gives period* 120. *Students who give the range as* $[-2, 2]$ *apply the amplitude to* $y = 0$ *instead of the midline* $y = 1$. *The range centres on the midline:* 1 ± 2.

Question 2 Hard – Paper 1 [6 marks]

Solve $2\cos(2x) - 1 = 0$ for $0 \le x \le 180$.

MISTAKE ANALYSIS

$\cos(2x) = \frac{1}{2}$. Let $u = 2x$, so $u \in [0, 360]$: $\cos u = \frac{1}{2} \Rightarrow u = 60$ or $u = 300$. $2x = 60 \Rightarrow x = 30$; $2x = 300 \Rightarrow x = 150$. Solutions: $x = 30, 150$. Students who solve $\cos x = \frac{1}{2}$ (forgetting the factor 2) get $x = 60$ only. Work with the full argument $2x$ and double the solution range to $[0, 360]$. Students who stop at $u = 60$ miss $u = 300$. Cosine is positive in both Q1 and Q4.

Question 3 Hard – Paper 2 [7 marks]

A triangle has sides $a = 5$, $b = 7$, $c = 9$. Find the area of the triangle, correct to 1 d.p.

MISTAKE ANALYSIS

Find angle C via cosine rule: $\cos C = \dfrac{25 + 49 - 81}{2(5)(7)} = \dfrac{-7}{70} = -0.1 \Rightarrow C = \cos^{-1}(-0.1) \approx 95.739$. Area $= \frac{1}{2}(5)(7)\sin(95.739) \approx \frac{1}{2}(35)(0.9950) \approx 17.4$ cm^2. Students who apply Heron's formula: $s = 10.5$, Area $= \sqrt{10.5 \times 5.5 \times 3.5 \times 1.5} = \sqrt{302.8125} \approx 17.4\checkmark$ – correct, but Heron's formula is not in the IB AA SL formula booklet. Use the cosine rule to find an angle, then $\frac{1}{2}ab\sin C$. Note: $C > 90$ because $c^2 = 81 > a^2 + b^2 = 74$. Sine of an obtuse angle is still positive.

Question 4 Medium – Paper 1 [6 marks]

Describe fully the transformation that maps $y = \sin x$ to $y = \sin(x - 30)$, and state the new x-intercepts nearest to $x = 0$.

MISTAKE ANALYSIS

$y = \sin(x - 30)$ is $y = \sin x$ translated 30 to the right (horizontal shift). $\sin x = 0$ at $x = 0, 180, \ldots$ so $\sin(x - 30) = 0$ at $x = 30, 210, \ldots$ The nearest x-intercept to the right of 0 is $x = 30$. Students who say the graph shifts left confuse $x - 30$ with $x + 30$. Inside the function, subtracting shifts right. This is the same direction-trap as all horizontal translations.

Question 5

Hard – Paper 2 [7 marks]

A ship leaves port A and sails 50 km on a bearing of 040 to reach B. It then sails 30 km on a bearing of 130 to reach C. Find the distance AC.

MISTAKE ANALYSIS

The angle at B between BA and BC: the bearing from B back to A is $040 + 180 = 220$. The bearing from B to C is 130. Angle $ABC = 220 - 130 = 90$ (or equivalently: the two travel directions are 40 from north and 130 from north; the supplement of their difference is $180 - (130 - 40) = 90$). Since $\angle B = 90$: $AC = \sqrt{50^2 + 30^2} = \sqrt{3400} = 10\sqrt{34} \approx 58.3$ km. Students who assume the angle between the bearings is $130 - 40 = 90$ directly are correct in result but must justify the calculation. The angle at B between the two sides of the triangle is found by comparing the bearing from B to A with the bearing from B to C.

Question 6

Hard – Paper 1 [6 marks]

Prove that $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = \dfrac{2}{\sin x}$.

MISTAKE ANALYSIS

Work on the left-hand side. Common denominator $\sin x(1 + \cos x)$:

$$LHS = \frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)} = \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1 + \cos x)}.$$

Since $\sin^2 x + \cos^2 x = 1$: numerator $= 1 + 1 + 2\cos x = 2(1 + \cos x)$.

$$LHS = \frac{2(1 + \cos x)}{\sin x(1 + \cos x)} = \frac{2}{\sin x}. \checkmark$$

Students who try to work from both sides simultaneously lose the mark for valid proof. Always begin with one side and transform it into the other. Students who expand $(1 + \cos x)^2$ as $1 + \cos^2 x$ (forgetting $2\cos x$) get the wrong numerator. $(1 + \cos x)^2 = 1 + 2\cos x + \cos^2 x$.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

Amplitude 2; period $360/3 = 120$; midline $y = 1$	*range* $[-1, 3]$	**M1 A1**

Solution – Question 2

$u = 2x \in [0, 360]$: $\cos u = \frac{1}{2}$; $u = 60, 300$	$x = 30, 150$	**M1 A1**

Solution – Question 3

$\cos C = \frac{-7}{70}$; $C \approx 95.7$; Area $= \frac{1}{2}(5)(7)\sin 95.7$	$17.4\ \text{cm}^2$	**M1 A1**

Solution – Question 4

Shift right 30; x-intercept nearest 0	$x = 30$	**M1 A1**

Solution – Question 5

Bearing $B \to A = 220$; $\angle B = 220 - 130 = 90$		**M1**
$AC = \sqrt{50^2 + 30^2} = \sqrt{3400}$	$10\sqrt{34} \approx 58.3$ km	**A1**

Solution – Question 6

Common denom: num $= \sin^2 x + (1+\cos x)^2 = 2+2\cos x$		**M1**
$\frac{2(1+\cos x)}{\sin x(1+\cos x)}$	$\dfrac{2}{\sin x}$ ✓	**M1 A1**