Trigonometry

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	36 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Area of triangle: Area $= \frac{1}{2}ab\sin C$ (two sides, included angle).

Ambiguous case: when using the sine rule to find an angle, there may be two valid triangles. Check whether B and $180 - B$ both satisfy the triangle angle sum.

Double angle formula: $\cos 2x = 1 - 2\sin^2 x$. Used when sin and $\cos(2x)$ appear together.

Arc and sector: arc $= r\theta$; sector area $= \frac{1}{2}r^2\theta$ (both with θ in radians).

Question 1

Medium – Paper 2 [5 marks]

A triangle has sides $a = 9$ cm and $b = 12$ cm with included angle $C = 72$. Find the area, correct to 2 d.p.

MISTAKE ANALYSIS

Area $= \frac{1}{2}(9)(12)\sin 72 = 54\sin 72 \approx 51.36$ *cm²*. *Students who use the cosine rule to find side c first and then use* $\frac{1}{2}bh$ *add unnecessary steps. The formula* $\frac{1}{2}ab\sin C$ *applies directly when two sides and the included angle are given. Ensure the calculator is in degree mode:* $\sin 72 \approx 0.9511$.

Question 2

Hard – Paper 1 [6 marks]

Solve $2\sin^2 x - \sin x - 1 = 0$ for $0 \leq x \leq 360$.

MISTAKE ANALYSIS

Factor: $(2\sin x+1)(\sin x-1)=0$, *so* $\sin x=-\frac{1}{2}$ *or* $\sin x=1$. $\sin x=1\Rightarrow x=90$. $\sin x=-\frac{1}{2}$*: reference angle* 30*, sine negative in Q3 and Q4:* $x=180+30=210$ *and* $x=360-30=330$*. Solutions:* $x=90$, 210, 330*. Students who attempt to rearrange without factorising typically miss a solution. Let* $u=\sin x$*:* $2u^2-u-1=(2u+1)(u-1)=0$ *is a standard quadratic. Students who solve* $u=1$ *only, or forget the quadratic has two roots, give incomplete answers.*

Question 3 Medium – Paper 2 [5 marks]

In triangle ABC, $a=6$, $b=8$, $c=10$. Find angle C and hence classify the triangle.

MISTAKE ANALYSIS

$\cos C=\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{36+64-100}{2(6)(8)}=\dfrac{0}{96}=0\Rightarrow C=90$. *Triangle is right-angled. Note:* $6^2+8^2=100=10^2$ *confirms it is a Pythagorean triple. Students who use the sine rule instead of the cosine rule when all three sides are given must first find a side-angle pair – slower. The cosine rule with all three sides directly gives any angle. Students who compute* $\frac{36+64-100}{96}$ *and write* $\frac{100}{96}\neq 0$ *must recheck:* $36+64=100$, *so the numerator is* $100-100=0$.

Question 4 Hard – Paper 2 [7 marks]

In triangle ABC, $a=7$ cm, $b=10$ cm, and $A=30$. Find all possible values of angle B, and find the corresponding values of side c. Give answers to 1 d.p.

MISTAKE ANALYSIS

Sine rule: $\sin B=\dfrac{b\sin A}{a}=\dfrac{10\sin 30}{7}=\dfrac{5}{7}\approx 0.7143$. $B_1=\sin^{-1}(5/7)\approx 45.6$. *Check:* $A+B_1=75.6<180\checkmark$. $C_1\approx 104.4$. $B_2=180-45.6=134.4$. *Check:* $A+B_2=164.4<180\checkmark$. $C_2\approx 15.6$. *Triangle 1:* $c_1=\dfrac{7\sin 104.4}{\sin 30}\approx 13.6$ *cm. Triangle 2:* $c_2=\dfrac{7\sin 15.6}{\sin 30}\approx 3.8$ *cm. Students who report only one triangle miss the ambiguous case. Whenever* $\sin B=k$ *gives* $B<90$*, check whether* $180-B$ *also gives a valid triangle.*

Question 5 Hard – Paper 1 [7 marks]

Solve $\sin x = \cos 2x$ for $0 \le x \le 360$.

MISTAKE ANALYSIS

Use $\cos 2x = 1 - 2\sin^2 x$*:* $\sin x = 1 - 2\sin^2 x \Rightarrow 2\sin^2 x + \sin x - 1 = 0 \Rightarrow (2\sin x - 1)(\sin x + 1) = 0$. $\sin x = \frac{1}{2}$*:* $x = 30, 150$. $\sin x = -1$*:* $x = 270$. *Solutions:* $x = 30, 150, 270$. *Students who substitute* $\cos 2x = \cos^2 x - \sin^2 x$ *reach a degree-3 equation with no clear factorisation. The identity* $\cos 2x = 1 - 2\sin^2 x$ *is the key choice because it expresses everything in* $\sin x$ *only.*

Question 6 Hard – Paper 1 [6 marks]

A sector of a circle has radius 8 cm and central angle $\dfrac{\pi}{4}$ radians. Find the arc length and the area of the sector, giving exact answers.

MISTAKE ANALYSIS

Arc length $= r\theta = 8 \times \dfrac{\pi}{4} = 2\pi$ *cm. Area* $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 64 \times \dfrac{\pi}{4} = 8\pi$ *cm²*. *Students who multiply the angle by* $\frac{180}{\pi}$ *to convert to degrees before applying the formulas introduce unnecessary steps. The arc and sector formulas require radians, and* $\frac{\pi}{4}$ *is already in radians. Students who confuse arc (*$r\theta$*) with area (*$\frac{1}{2}r^2\theta$*) often apply the area formula to find the arc, or omit the* $\frac{1}{2}$*. Keep the two formulas distinct.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$\frac{1}{2}(9)(12)\sin 72$ 51.36 cm^2 **M1 A1**

Solution – Question 2

$(2\sin x + 1)(\sin x - 1) = 0$; $\sin x = -\frac{1}{2}$ or $\sin x = 1$ **M1**

$\sin x = 1 \Rightarrow x = 90$; $\sin x = -\frac{1}{2} \Rightarrow x = 210, 330$ $x = 90, 210, 330$ **A1**

Solution – Question 3

$\cos C = \frac{36+64-100}{96} = 0$; right-angled Pythagorean triple $C = 90$ **M1 A1**

Solution – Question 4

$\sin B = 5/7$; $B_1 \approx 45.6$, $B_2 \approx 134.4$ (both valid) **M1 A1**

$c_1 \approx 13.6$ cm; $c_2 \approx 3.8$ cm **M1 A1 R1**

Solution – Question 5

$\cos 2x = 1 - 2\sin^2 x$: $(2\sin x - 1)(\sin x + 1) = 0$ $x = 30, 150, 270$ **M1 A1**

Solution – Question 6

Arc $= 8 \times \frac{\pi}{4}$ $= 2\pi$ cm; Area $= \frac{1}{2}(64)\frac{\pi}{4}$ $= 8\pi$ cm^2 **M1 A1**

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