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Trigonometry

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	34 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Exact values: know the unit circle. $\sin 30 = \frac{1}{2}$, $\cos 30 = \frac{\sqrt{3}}{2}$, $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$.

Solving in a range: a single equation like $\sin x = k$ usually has two solutions per 360. Use the symmetry of the unit circle.

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$. **Cosine rule:** $c^2 = a^2 + b^2 - 2ab\cos C$.

Identity: $\sin^2\theta + \cos^2\theta = 1$.

Question 1

Medium – Paper 1 [6 marks]

Find the exact value of each of the following:

(a) $\sin 120$

(b) $\cos 135$

(c) $\tan 225$

MISTAKE ANALYSIS

(a) 120 is in the second quadrant where sine is positive. Reference angle $180 - 120 = 60$: $\sin 120 = \sin 60 = \dfrac{\sqrt{3}}{2}$. (b) 135 is in the second quadrant where cosine is negative. Reference angle 45: $\cos 135 = -\cos 45 = -\dfrac{\sqrt{2}}{2}$. (c) 225 is in the third quadrant where tangent is positive. Reference angle $225 - 180 = 45$: $\tan 225 = \tan 45 = 1$. Students who ignore the quadrant sign give $\cos 135 = +\frac{\sqrt{2}}{2}$. Use the CAST rule: in the second quadrant only sine is positive, so cosine and tangent are negative there.

Question 2

Medium – Paper 1 [5 marks]

Solve $\sin x = \dfrac{1}{2}$ for $0 \le x \le 360$.

MISTAKE ANALYSIS

The principal value is $x = 30$. Sine is also positive in the second quadrant: $x = 180 - 30 = 150$. Solutions: $x = 30$ and $x = 150$. Students who give only $x = 30$ miss the second-quadrant solution. The equation $\sin x = \frac{1}{2}$ has two solutions in $[0, 360]$ because sine is positive in both the first and second quadrants.

Question 3

Medium – Paper 1 [5 marks]

Solve $2\cos x + 1 = 0$ for $0 \le x \le 360$.

MISTAKE ANALYSIS

$2\cos x + 1 = 0 \Rightarrow \cos x = -\dfrac{1}{2}$. Reference angle: $\cos^{-1}\frac{1}{2} = 60$. Cosine is negative in the second and third quadrants: $x = 180 - 60 = 120$ and $x = 180 + 60 = 240$. Students who solve $\cos x = -\frac{1}{2}$ and give $x = 60$ ignore the negative sign and the quadrants. Cosine is negative in quadrants 2 and 3, giving 120 and 240.

Question 4

Medium – Paper 1 [5 marks]

Show that $\dfrac{1 - \cos^2 x}{\sin x} = \sin x$ for $\sin x \ne 0$.

MISTAKE ANALYSIS

Using the identity $\sin^2 x + \cos^2 x = 1$, we have $1 - \cos^2 x = \sin^2 x$. So $\dfrac{1-\cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} = \sin x$. ✓ Students who try to expand $1 - \cos^2 x$ in other ways overcomplicate it. The Pythagorean identity rearranges directly to $1 - \cos^2 x = \sin^2 x$, which cancels with the denominator.

Question 5 Medium – Paper 2 [6 marks]

In triangle ABC, $a = 8$ cm, angle $A = 40$, and angle $B = 65$. Find the length of side b, correct to 2 d.p.

MISTAKE ANALYSIS

By the sine rule: $\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow b = \frac{a \sin B}{\sin A} = \frac{8 \sin 65}{\sin 40} \approx 11.28$ *cm. Students who write* $b = \frac{a \sin A}{\sin B}$ *invert the ratio. The side and its opposite angle stay together:* b *pairs with* $\sin B$*,* a *pairs with* $\sin A$*. Solve* $\frac{b}{\sin B} = \frac{a}{\sin A}$ *for* b*. Check the calculator is in degree mode, not radians.*

Question 6 Medium – Paper 2 [7 marks]

In triangle ABC, $a = 7$ cm, $b = 10$ cm, and the included angle $C = 50$. Find the length of side c, correct to 2 d.p.

MISTAKE ANALYSIS

By the cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C = 7^2 + 10^2 - 2(7)(10) \cos 50 = 49 + 100 - 140 \cos 50 \approx 59.0$. $c \approx \sqrt{59.0} \approx 7.68$ *cm. Students who forget to take the square root report* $c \approx 59.0$*. The cosine rule gives* c^2*; the final step is* $c = \sqrt{c^2}$*. Students who use* $+2ab \cos C$ *instead of* $-2ab \cos C$ *get the sign wrong. The cosine rule subtracts the* $2ab \cos C$ *term.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

(a) Q2, ref 60	$\sin 120 = \frac{\sqrt{3}}{2}$	**A1**
(b) Q2, ref 45, cosine negative	$\cos 135 = -\frac{\sqrt{2}}{2}$	**A1**
(c) Q3, ref 45, tangent positive	$\tan 225 = 1$	**A1**

Solution – Question 2

Principal 30; second quadrant 180 − 30	$x = 30,\ 150$	**M1 A1**

Solution – Question 3

$\cos x = -\frac{1}{2}$; ref 60; Q2 and Q3	$x = 120,\ 240$	**M1 A1**

Solution – Question 4

$1 - \cos^2 x = \sin^2 x$; $\frac{\sin^2 x}{\sin x}$	$\sin x$	**M1 A1**

Solution – Question 5

$b = \frac{8 \sin 65}{\sin 40}$	11.28 cm	**M1 A1**

Solution – Question 6

$c^2 = 49 + 100 - 140\cos 50 \approx 59.0$ **M1**

$c = \sqrt{59.0}$ 7.68 cm **A1**

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