Statistics & Probability

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 4 – Statistics & Probability
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	34 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Addition law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Independence: A and B are independent if and only if $P(A \cap B) = P(A) \times P(B)$.

Binomial: $E(X) = np$; $\text{Var}(X) = np(1-p)$.

Normal – inverse: given $P(X < x) = p$, find z from the standard normal tables, then $x = \mu + z\sigma$.

Variance: $\text{Var}(X) = E(X^2) - [E(X)]^2$.

Question 1

Medium – Paper 1 [4 marks]

Events A and B are such that $P(A) = 0.4$, $P(B) = 0.6$, and $P(A \cap B) = 0.1$. Find $P(A \cup B)$.

MISTAKE ANALYSIS

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.6 - 0.1 = 0.9$. *Students who write* $P(A \cup B) = P(A) + P(B) = 1.0$ *apply the addition law for mutually exclusive events. The events are not mutually exclusive since* $P(A \cap B) = 0.1 \neq 0$. *The intersection must be subtracted to avoid double-counting.*

Question 2

Medium – Paper 1 [5 marks]

Using the values in Question 1, determine whether A and B are independent. Give a reason.

MISTAKE ANALYSIS

For independence: $P(A) \times P(B) = 0.4 \times 0.6 = 0.24$. But $P(A \cap B) = 0.1 \neq 0.24$. Since $P(A \cap B) \neq P(A) \times P(B)$, the events are ***not*** *independent. Students who check $P(A \cup B) = 1$ (a condition for mutually exclusive complementary events, not independence) confuse independence with mutual exclusivity. Independence requires $P(A \cap B) = P(A)P(B)$.*

Question 3 Medium – Paper 2 [5 marks]

$X \sim B(20, 0.4)$. Find $E(X)$, $\text{Var}(X)$, and the standard deviation of X.

MISTAKE ANALYSIS

$E(X) = np = 20 \times 0.4 = 8$. $Var(X) = np(1-p) = 20 \times 0.4 \times 0.6 = 4.8$. $SD(X) = \sqrt{4.8} \approx 2.19$. Students who write $Var(X) = np^2 = 20 \times 0.16 = 3.2$ use p^2 instead of $p(1-p)$. Students who write $SD = 4.8$ report the variance as the standard deviation. $SD = \sqrt{Var}$.

Question 4 Hard – Paper 2 [5 marks]

Heights are normally distributed with mean 165 cm and standard deviation 8 cm. Find the probability that a randomly selected height exceeds 175 cm.

MISTAKE ANALYSIS

$Z = \dfrac{175 - 165}{8} = 1.25$. $P(X > 175) = P(Z > 1.25) = 1 - P(Z < 1.25) \approx 1 - 0.8944 = 0.1056$. *Students who write $P(X > 175) = P(Z > 1.25) = 0.8944$ give the lower tail $P(Z < 1.25)$ instead of the upper tail $P(Z > 1.25) = 1 - 0.8944$. The question asks for heights that* exceed *175 cm – the upper tail.*

Question 5 Hard – Paper 2 [7 marks]

$X \sim N(\mu, 10^2)$. Given that $P(X < 50) = 0.25$, find μ.

MISTAKE ANALYSIS

$P(X < 50) = 0.25 \Rightarrow P(Z < z) = 0.25 \Rightarrow z = -0.6745$ (lower tail, negative z). $50 = \mu + (-0.6745) \times 10 \Rightarrow \mu = 50 + 6.745 \approx 56.74$ (or 56.75 depending on table precision). Students who use $z = +0.6745$ (positive, ignoring that 0.25 is in the lower tail) get $\mu \approx 43.26$. Since $P(X < 50) = 0.25 < 0.5$, the value 50 is below the mean: $\mu > 50$.

Question 6 Hard – Paper 1 [8 marks]

A discrete random variable X has the following distribution:

x	1	2	3
$P(X = x)$	0.3	0.5	0.2

Find $E(X)$ and $\text{Var}(X)$.

MISTAKE ANALYSIS

$E(X) = 1(0.3) + 2(0.5) + 3(0.2) = 0.3 + 1.0 + 0.6 = 1.9$. $E(X^2) = 1^2(0.3) + 2^2(0.5) + 3^2(0.2) = 0.3 + 2.0 + 1.8 = 4.1$. $Var(X) = E(X^2) - [E(X)]^2 = 4.1 - 1.9^2 = 4.1 - 3.61 = 0.49$. Students who write $Var(X) = E(X^2 - [E(X)]^2) = E(X^2) - E([E(X)]^2)$ confuse inside and outside the expectation. The formula is $E(X^2) - [E(X)]^2$: compute $E(X)$ first, square it, then subtract from $E(X^2)$.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$0.4 + 0.6 - 0.1$		0.9	**M1 A1**

Solution – Question 2

$P(A)P(B)$	$=$	not independent	**M1 A1 R1**
0.24	$\neq$		
$P(A \cap B) = 0.1$			

Solution – Question 3

$E = 8$; Var $= 4.8$; SD $= \sqrt{4.8}$		$E(X) = 8,\ \text{Var} = 4.8,\ \text{SD} \approx 2.19$	**M1 A1**

Solution – Question 4

$Z > 1.25$; $P(Z > 1.25) = 1 - 0.8944$		0.1056	**M1 A1**

Solution – Question 5

$z = -0.6745$; $\mu = 50 + 6.745$		$\mu \approx 56.74$	**M1 A1**

Solution – Question 6

$E(X) = 1.9$; $E(X^2) = 4.1$; Var $= 4.1 - 3.61$		$E(X) = 1.9,\ \text{Var}(X) = 0.49$	**M1 A1**