Statistics & Probability

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 4 – Statistics & Probability
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	33 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Basic probability: $P(A) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$. $P(A') = 1 - P(A)$.

Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Binomial distribution: $X \sim B(n, p)$. $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Normal distribution: standardise $Z = \dfrac{X - \mu}{\sigma}$ then use the standard normal tables or GDC.

Expected value: $E(X) = \sum x_i P(X = x_i)$.

Question 1

Medium – Paper 1 [4 marks]

A bag contains 5 red, 3 blue, and 2 green balls. One ball is selected at random.

(a) Find the probability that the ball is red.

(b) Find the probability that the ball is not red.

MISTAKE ANALYSIS

(a) $P(\text{red}) = \dfrac{5}{10} = \dfrac{1}{2}$. *(b)* $P(\text{not red}) = 1 - \dfrac{1}{2} = \dfrac{1}{2}$. *Students who write* $P(\text{not red}) = \dfrac{5}{10}$ *use the number of red balls instead of the number of non-red balls (3+2=5). Alternatively, the complement rule:* $1 - P(\text{red}) = \frac{1}{2}$.

Question 2 Medium – Paper 1 [5 marks]

Two fair six-sided dice are rolled. Find the probability that the sum of the two dice is 7.

MISTAKE ANALYSIS

Total outcomes: $6 \times 6 = 36$. *Favourable:* $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1) = 6$ *pairs.* $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$. *Students who write* $P = \frac{7}{36}$ *list the number 7 rather than counting the pairs that sum to 7. Students who write* $P = \frac{6}{12}$ *halve the total incorrectly – total outcomes for two dice is* $6^2 = 36$.

Question 3 Medium – Paper 1 [5 marks]

Events A and B are such that $P(A \cap B) = 0.2$ and $P(B) = 0.5$. Find $P(A \mid B)$.

MISTAKE ANALYSIS

$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4$. *Students who write* $P(A \mid B) = \frac{P(B)}{P(A \cap B)} = \frac{0.5}{0.2} = 2.5$ *invert the formula. A probability cannot exceed 1 – this should immediately signal an error. Students who write* $P(A \mid B) = P(A \cap B) \times P(B) = 0.1$ *multiply instead of divide.*

Question 4 Hard – Paper 2 [7 marks]

A random variable $X \sim B(10,\, 0.3)$. Find $P(X = 4)$, correct to 4 significant figures.

MISTAKE ANALYSIS

$P(X = 4) = \binom{10}{4}(0.3)^4(0.7)^6 = 210 \times 0.0081 \times 0.117649 \approx 0.2001$. *Students who omit the binomial coefficient* $\binom{10}{4} = 210$ *write* $(0.3)^4(0.7)^6 \approx 0.000953$ *– far too small. Students who use* $\binom{10}{4} = \binom{10}{6}$ *must verify:* $\binom{10}{4} = \frac{10!}{4!6!} = 210$ ✓. *Ensure the GDC is set to binomial probability, not cumulative.*

Question 5 Hard – Paper 2 [5 marks]

The heights of students at a school are normally distributed with mean 165 cm and standard deviation 8 cm. Find the probability that a randomly selected student has height less than 173 cm.

MISTAKE ANALYSIS

$Z = \dfrac{173 - 165}{8} = 1$. $P(X < 173) = P(Z < 1) \approx 0.8413$. *Students who write* $P(Z < 1) = 1 - P(Z < 1) = 0.1587$ *find the upper tail* $P(X > 173)$ *instead of the lower tail. Students who standardise to* $Z = \frac{173}{165} \approx 1.05$ *use* μ *as a divisor instead of computing* $\frac{x-\mu}{\sigma}$.

Question 6 Hard – Paper 1 [7 marks]

A discrete random variable X has the probability distribution:

x	0	1	2
$P(X = x)$	0.2	0.5	0.3

Find $E(X)$.

MISTAKE ANALYSIS

$E(X) = 0 \times 0.2 + 1 \times 0.5 + 2 \times 0.3 = 0 + 0.5 + 0.6 = 1.1$. *Students who write* $E(X) = \dfrac{0+1+2}{3} = 1$ *compute the mean of the* x*-values, ignoring the probabilities. The expected value weights each value by its probability. Students who omit the* $x = 0$ *term:* $1(0.5) + 2(0.3) = 1.1$ *– correct by accident here, but the* $0 \times 0.2 = 0$ *term must be included in the method.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

(a) $5/10$; (b) $1 - 1/2$	$\frac{1}{2}$; $\frac{1}{2}$	**M1 A1**

Solution – Question 2

6 favourable pairs; total 36	$\frac{1}{6}$	**M1 A1**

Solution – Question 3

$P(A\|B) = P(A \cap B)/P(B) = 0.2/0.5$	$= 0.4$	**M1 A1**

Solution – Question 4

$\binom{10}{4}(0.3)^4(0.7)^6 = 210 \times \cdots$	0.2001	**M1 A1**

Solution – Question 5

$Z = (173 - 165)/8 = 1$; $P(Z < 1)$	$= 0.8413$	**M1 A1**

Solution – Question 6

$0(0.2) + 1(0.5) + 2(0.3)$	1.1	**M1 A1**