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Sequences & Series

Mistake Analysis – Set III

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 1 – Number & Algebra
Level	Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	37 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Sum of integers: $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$.

Minimum n from inequality: solve $S_n > k$ for n using logarithms, then round up to the next integer.

Term from partial sum: $T_n = S_n - S_{n-1}$ for $n \geq 2$; check $n = 1$ separately.

Infinite GP with sigma: identify the first term and common ratio from the general term, then apply $S_\infty = \frac{a}{1-r}$.

Question 1 — Hard – Paper 2 — [5 marks]

Find the sum of all integers from 50 to 150 inclusive.

MISTAKE ANALYSIS

This is an AP with $a = 50$, $l = 150$, and $n = 150 - 50 + 1 = 101$ terms. $S = \frac{n}{2}(a + l) = \frac{101}{2}(50 + 150) = \frac{101 \times 200}{2} = 10100$. Students who compute $n = 150 - 50 = 100$ terms miss the "inclusive" condition. Both endpoints are included: $n = 150 - 50 + 1 = 101$.

Question 2 — Hard – Paper 2 — [6 marks]

A geometric series has first term 5 and common ratio 2. Find the smallest value of n for which $S_n > 1000$.

MISTAKE ANALYSIS

$S_n = \dfrac{5(2^n - 1)}{2 - 1} = 5(2^n - 1) > 1000 \Rightarrow 2^n - 1 > 200 \Rightarrow 2^n > 201$. *Taking logarithms:* $n > \dfrac{\ln 201}{\ln 2} \approx 7.651$. *Since n must be a positive integer, $n = 8$. Check:* $S_8 = 5(256 - 1) = 1275 > 1000\,\checkmark$; $S_7 = 5(128 - 1) = 635 < 1000\,\checkmark$. *Students who round 7.651 down to $n = 7$ choose the value for which $S_n < 1000$ – the question asks for the sum to* exceed *1000, which requires rounding up.*

Question 3

Hard – Paper 1 [6 marks]

Find the value of $\displaystyle\sum_{r=1}^{20}(5r - 2)$.

MISTAKE ANALYSIS

$\displaystyle\sum_{r=1}^{20}(5r - 2) = 5\sum_{r=1}^{20} r - 2 \times 20 = 5 \times \frac{20 \times 21}{2} - 40 = 5 \times 210 - 40 = 1050 - 40 = 1010$. *Students who write* $5 \times \frac{20\times21}{2} - 2 = 1050 - 2 = 1048$ *subtract the constant term only once. The constant -2 must be subtracted for each of the 20 values of r:* $-2 \times 20 = -40$.

Question 4

Hard – Paper 1 [7 marks]

The sum of the first n terms of a sequence is $S_n = 3n^2 - n$.

(a) Find T_n for $n \geq 2$.

(b) Show that the sequence is arithmetic and state the common difference.

MISTAKE ANALYSIS

(a) $T_n = S_n - S_{n-1} = (3n^2 - n) - (3(n-1)^2 - (n-1))$. $= (3n^2 - n) - (3n^2 - 6n + 3 - n + 1) = (3n^2 - n) - (3n^2 - 7n + 4) = 6n - 4$. *Check $n = 1$:* $T_1 = S_1 = 3 - 1 = 2$ *and* $6(1) - 4 = 2\,\checkmark$. *So* $T_n = 6n - 4$ *for all* $n \geq 1$. *(b)* $T_{n+1} - T_n = (6(n+1) - 4) - (6n - 4) = 6$. *Constant difference $d = 6$: arithmetic* $\checkmark$. *Students who skip the $n = 1$ check:* $T_n = S_n - S_{n-1}$ *is only valid for* $n \geq 2$. *Always verify* $T_1 = S_1$ *separately.*

Question 5 Hard – Paper 2 [6 marks]

An investment of \$10 000 grows at 5% per year. Find the smallest number of complete years for the investment to exceed \$13 000.

MISTAKE ANALYSIS

After n years: $10000 \times (1.05)^n > 13000 \Rightarrow (1.05)^n > 1.3$. $n > \dfrac{\ln 1.3}{\ln 1.05} \approx \dfrac{0.2624}{0.04879} \approx 5.378$. *Smallest integer:* $n = 6$. *Check:* $10000 \times (1.05)^6 \approx 13401 > 13000\,\checkmark$; $10000 \times (1.05)^5 \approx 12763 < 13000\,\checkmark$. *Students who answer $n = 5$ round down from 5.378. The investment only exceeds \$13 000 after 6 complete years. Always round up for "smallest n" questions involving strict inequalities.*

Question 6 Hard – Paper 1 [7 marks]

Find the exact value of $\displaystyle\sum_{n=1}^{\infty} 12\left(\frac{2}{3}\right)^n$.

MISTAKE ANALYSIS

The first term (at $n = 1$) is $12 \times \dfrac{2}{3} = 8$. *The common ratio is* $r = \dfrac{2}{3}$. *Since* $|r| = \dfrac{2}{3} < 1$, *the series converges.* $S_\infty = \dfrac{8}{1-\frac{2}{3}} = \dfrac{8}{\frac{1}{3}} = 24$. *Students who use $a = 12$ (the coefficient in front of the bracket) as the first term get* $S_\infty = \frac{12}{1/3} = 36$. *The first term is found by substituting $n = 1$:* $a = 12 \times \frac{2}{3} = 8$, *not 12.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$n = 101$; $S = \frac{101}{2}(50 + 150) = \frac{101 \times 200}{2}$	10100	**M1 A1**

Solution – Question 2

$2^n > 201$; $n > \frac{\ln 201}{\ln 2} \approx 7.651$; round up	$n = 8$	**M1 A1 R1**

Solution – Question 3

$5 \times 210 - 2 \times 20 = 1050 - 40$	1010	**M1 A1**

Solution – Question 4

(a) $T_n = (3n^2 - n) - (3n^2 - 7n + 4)$	$T_n = 6n - 4$	**M1 A1**
(b) $T_{n+1} - T_n = 6$ (constant)	$d = 6,$ arithmetic	**M1 R1**

Solution – Question 5

$(1.05)^n > 1.3$; $n > \frac{\ln 1.3}{\ln 1.05} \approx 5.38$; round up	$n = 6$ years	**M1 A1 R1**

Solution – Question 6

First term $= 12 \times \frac{2}{3} = 8$; $r = \frac{2}{3}$; $S_\infty = \frac{8}{1/3}$	24	**M1 A1**

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