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Sequences & Series

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 1 – Number & Algebra
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	34 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Simultaneous equations for AP/GP: when two terms are given, set up two equations and solve.

Sigma notation: $\sum_{r=1}^{n}(ar+b) = a \cdot \frac{n(n+1)}{2} + bn$.

Term from partial sum: $T_n = S_n - S_{n-1}$ for $n \geq 2$, and $T_1 = S_1$.

Convergence: state $|r| < 1$ and compute r from the given information before applying S_∞.

Question 1

Medium – Paper 1 [5 marks]

An arithmetic sequence has 5th term 23 and 10th term 43. Find the first term and common difference.

MISTAKE ANALYSIS

$T_5 = a + 4d = 23$ and $T_{10} = a + 9d = 43$. Subtract: $5d = 20 \Rightarrow d = 4$. Then $a = 23 - 4(4) = 23 - 16 = 7$. Students who write $T_5 = 5a + d = 23$ misplace the coefficient. The formula is $a + (n-1)d$, so $T_5 = a + 4d$, not $5a + d$.

Question 2

Medium – Paper 1 [5 marks]

A geometric sequence has 3rd term 12 and 6th term 96. Find the first term and common ratio.

MISTAKE ANALYSIS

$T_3 = ar^2 = 12$ and $T_6 = ar^5 = 96$. Divide: $r^3 = \frac{96}{12} = 8 \Rightarrow r = 2$. Then $a = \frac{12}{r^2} = \frac{12}{4} = 3$. Students who subtract the equations (as for AP) cannot solve the resulting expression. For GP, divide consecutive term equations to eliminate a: the ratio of T_6 to T_3 is r^3.

Question 3 Medium – Paper 1 [5 marks]

Find the value of $\sum_{r=1}^{10}(3r + 1)$.

MISTAKE ANALYSIS

$\sum_{r=1}^{10}(3r + 1) = 3\sum_{r=1}^{10} r + \sum_{r=1}^{10} 1 = 3 \cdot \frac{10 \times 11}{2} + 10 \times 1 = 165 + 10 = 175$. *Students who compute* $3(1 + 2 + \cdots + 10) + 1 = 165 + 1 = 166$ *apply the* $+1$ *only once instead of 10 times. The constant term* 1 *is added for each of the 10 values of* r.

Question 4 Hard – Paper 1 [6 marks]

The sum of the first n terms of a sequence is given by $S_n = n^2 + 3n$. Find an expression for the nth term T_n, and show that the sequence is arithmetic.

MISTAKE ANALYSIS

For $n \geq 2$: $T_n = S_n - S_{n-1} = (n^2 + 3n) - ((n-1)^2 + 3(n-1))$. $= (n^2 + 3n) - (n^2 - 2n + 1 + 3n - 3) = (n^2 + 3n) - (n^2 + n - 2) = 2n + 2$. Check $n = 1$: $T_1 = S_1 = 1 + 3 = 4 = 2(1) + 2\checkmark$. Since $T_n = 2n + 2$ is linear in n, the sequence is arithmetic with $d = 2$. Students who differentiate S_n with respect to n are applying calculus to a discrete sequence incorrectly. The correct method is $T_n = S_n - S_{n-1}$.

Question 5 Hard – Paper 1 [6 marks]

A geometric series has first term a and common ratio $\dfrac{x}{3}$. State the values of x for which the sum to infinity exists, giving your answer as an inequality.

MISTAKE ANALYSIS

S_∞ exists when $\left|\dfrac{x}{3}\right| < 1 \Rightarrow |x| < 3 \Rightarrow -3 < x < 3$. Students who write $\frac{x}{3} < 1 \Rightarrow x < 3$ (only the upper bound) forget that the modulus gives two conditions. The ratio must satisfy $-1 < \frac{x}{3} < 1$, which gives both $x < 3$ and $x > -3$.

Question 6 Hard – Paper 1 [7 marks]

Three numbers are in geometric progression. The first is 3, the last is 12, and the middle term is x. Find x and hence find the sum of the first 6 terms of the GP.

MISTAKE ANALYSIS

In a GP, the middle term satisfies $x^2 = 3 \times 12 = 36 \Rightarrow x = 6$ (taking the positive value). Common ratio $r = \dfrac{6}{3} = 2$. Sum of first 6 terms: $S_6 = \dfrac{3(2^6 - 1)}{2 - 1} = \dfrac{3 \times 63}{1} = 189$. Students who take $x = \pm 6$: both are valid solutions ($r = 2$ or $r = -2$). If $x = -6$ then $r = -2$ and $S_6 = \dfrac{3((-2)^6 - 1)}{-2 - 1} = \dfrac{3 \times 63}{-3} = -63$. The question says "the last is 12" – since $3 \times (-2)^2 = 12$, both are valid. Accept either.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$5d = 20 \Rightarrow d = 4; a = 23 - 16$	$a = 7,\ d = 4$	**M1 A1**

Solution – Question 2

$r^3 = 96/12 = 8 \Rightarrow r = 2; a = 12/4$	$a = 3,\ r = 2$	**M1 A1**

Solution – Question 3

$3 \times \frac{10\times 11}{2} + 10 \times 1 = 165 + 10$	175	**M1 A1**

Solution – Question 4

$T_n = S_n - S_{n-1} = (n^2 + 3n) - (n^2 + n - 2) = 2n + 2$		**M1 A1**
Linear in n with constant difference 2	arithmetic, $d = 2$	**R1**

Solution – Question 5

$\left\lvert \frac{x}{3} \right\rvert < 1$	$-3 < x < 3$	**M1 A1**

Solution – Question 6

$x^2 = 36 \Rightarrow x = 6; r = 2; S_6 = 3(64 - 1)/1$	189	**M1 A1**

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