

Sequences & Series

Mistake Analysis – Set I

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| Course | IB Mathematics: Analysis & Approaches SL |
| Topic | Topic 1 – Number & Algebra |
| Level | Medium → Hard (Paper 1 and Paper 2 style) |
| Questions | 6 |
| Marks | 31 total. M1 method · A1 accuracy · R1 reasoning. |

BEFORE YOU BEGIN

Arithmetic progression (AP): $T_n = a + (n - 1)d$. $S_n = \frac{n}{2}(2a + (n - 1)d) = \frac{n}{2}(a + l)$.

Geometric progression (GP): $T_n = ar^{n-1}$. $S_n = \frac{a(r^n - 1)}{r - 1}$ for $r \neq 1$.

Sum to infinity: $S_\infty = \frac{a}{1 - r}$, valid only when $|r| < 1$.

Key discipline: identify whether the sequence is AP or GP before applying any formula.

Question 1

Medium – Paper 1

[4 marks]

An arithmetic sequence has first term 3 and common difference 4. Find the 20th term.

MISTAKE ANALYSIS

$T_{20} = a + (n - 1)d = 3 + 19 \times 4 = 3 + 76 = 79$. *Students who write $T_{20} = 3 + 20 \times 4 = 83$ use n instead of $(n - 1)$ in the formula. The formula starts counting from 0: the first term needs 0 steps of d , not 1. So T_n requires $(n - 1)$ steps.*

Question 2

Medium – Paper 1

[5 marks]

An arithmetic sequence has first term 5 and common difference 3. Find the sum of the first 15 terms.

MISTAKE ANALYSIS

$S_{15} = \frac{15}{2}(2 \times 5 + (15 - 1) \times 3) = \frac{15}{2}(10 + 42) = \frac{15}{2} \times 52 = 390$. Students who use $S_n = \frac{n}{2}(a + l)$ without first finding the last term $l = T_{15} = 5 + 14 \times 3 = 47$ must use the formula with $2a + (n - 1)d$. Alternatively, $\frac{15}{2}(5 + 47) = \frac{15}{2} \times 52 = 390 \checkmark$. Both routes are correct.

Question 3

Medium – Paper 1

[4 marks]

A geometric sequence has first term 2 and common ratio 3. Find the 8th term.

MISTAKE ANALYSIS

$T_8 = ar^{n-1} = 2 \times 3^7 = 2 \times 2187 = 4374$. Students who write $T_8 = 2 \times 3^8 = 13122$ use r^n instead of r^{n-1} . The first term needs $r^0 = 1$ (zero multiplications by r), so the n th term needs r^{n-1} .

Question 4

Medium – Paper 1

[5 marks]

A geometric sequence has first term 8 and common ratio $\frac{1}{2}$. Find the sum to infinity.

MISTAKE ANALYSIS

$|r| = \frac{1}{2} < 1$, so the series converges. $S_\infty = \frac{a}{1 - r} = \frac{8}{1 - \frac{1}{2}} = \frac{8}{\frac{1}{2}} = 16$. Students who apply the formula without checking $|r| < 1$ may attempt S_∞ even for divergent series. Always state the convergence condition before applying the formula. The reasoning mark (R1) is awarded for this justification.

Question 5

Hard – Paper 1

[6 marks]

The n th term of an arithmetic sequence is given by $T_n = 5 + (n - 1) \times 3$. Find the value of n for which $T_n = 65$.

MISTAKE ANALYSIS

$5 + (n - 1) \times 3 = 65 \Rightarrow (n - 1) \times 3 = 60 \Rightarrow n - 1 = 20 \Rightarrow n = 21$. Check: $T_{21} = 5 + 20 \times 3 = 65 \checkmark$. Students who solve $5 + 3n = 65 \Rightarrow n = 20$ forget that the formula uses $(n - 1)$, not n . Always substitute back to verify.

Question 6

Hard – Paper 2

[7 marks]

A geometric sequence has first term 3 and common ratio 2. Find the sum of the first 6 terms.

MISTAKE ANALYSIS

$r = 2 \neq 1$, so use $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{3(2^6 - 1)}{2 - 1} = \frac{3 \times 63}{1} = 189$. Students who use $S_n = \frac{a(1 - r^n)}{1 - r}$ with $r = 2$: $\frac{3(1 - 64)}{1 - 2} = \frac{3(-63)}{-1} = 189 \checkmark$ – both forms give the same answer. The confusion arises when students mix signs carelessly: $\frac{-189}{-1} = 189$, not -189 .

WORKED SOLUTIONS – SET I – SEQUENCES & SERIES

M1 method · A1 accuracy · R1 reasoning

Solution – Question 1

$$T_{20} = 3 + (20 - 1) \times 4 = 3 + 76$$

M1 A1

Solution – Question 2

$$S_{15} = \frac{15}{2}(2 \times 5 + 14 \times 3) = \frac{15}{2} \times 52$$

M1 A1

Solution – Question 3

$$T_8 = 2 \times 3^7 = 2 \times 2187$$

M1 A1

Solution – Question 4

$$|r| = \frac{1}{2} < 1 \text{ (converges); } S_{\infty} = \frac{8}{1 - \frac{1}{2}}$$

M1 A1 R1

Solution – Question 5

$$(n - 1) \times 3 = 60 \Rightarrow n - 1 = 20$$

M1 A1

Solution – Question 6

$$S_6 = \frac{3(2^6 - 1)}{2 - 1} = \frac{3 \times 63}{1} = 189$$

M1 A1
