THE SANCTUARY OF LEARNING – ABDULWADOOD.ORG

Integration

Mistake Analysis – Set III

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	40 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Integration by parts: $\int u\,dv = uv - \int v\,du$. Choose u using LIATE (Logarithm, Inverse trig, Algebraic, Trig, Exponential).

Trig identity: $\cos^2 x = \dfrac{1+\cos(2x)}{2}$; $\sin^2 x = \dfrac{1-\cos(2x)}{2}$. Use these before integrating squared trig functions.

Cyclic by parts: when the original integral reappears, collect it on one side and divide.

Question 1

Hard – Paper 1 [7 marks]

Find $\int e^x \sin x\,dx$.

MISTAKE ANALYSIS

Let $u = \sin x$, $dv = e^x\,dx$: $v = e^x$, $du = \cos x\,dx$. $\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$. For $\int e^x \cos x\,dx$: let $u = \cos x$, $dv = e^x\,dx$: $= e^x \cos x + \int e^x \sin x\,dx$. Let $I = \int e^x \sin x\,dx$: $I = e^x \sin x - e^x \cos x - I \Rightarrow 2I = e^x(\sin x - \cos x) \Rightarrow I = \dfrac{e^x(\sin x - \cos x)}{2} + C$. Students who apply by parts once and stop have an incomplete answer. The cyclic case requires two applications, then collecting I from both sides.

Question 2

Hard – Paper 1 [6 marks]

Find $\int xe^x \, dx$.

MISTAKE ANALYSIS

By parts: $u = x$, $dv = e^x \, dx$, *so* $du = dx$, $v = e^x$. $\displaystyle\int xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + C = e^x(x-1) + C$. *Students who choose* $u = e^x$ *and* $dv = x \, dx$: $v = \frac{x^2}{2}$, *giving* $\frac{x^2 e^x}{2} - \int \frac{x^2}{2} e^x \, dx$ *– this is harder, not simpler. LIATE says* $u = x$ *(algebraic) before* e^x *(exponential).*

Question 3 Hard – Paper 1 [6 marks]

Find $\int x \ln x \, dx$.

MISTAKE ANALYSIS

LIATE: $u = \ln x$, $dv = x \, dx$, *so* $du = \dfrac{1}{x} dx$, $v = \dfrac{x^2}{2}$. $\displaystyle\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx = \frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C = \frac{x^2(2\ln x - 1)}{4} + C$. *Students who choose* $u = x$ *and* $dv = \ln x \, dx$: $\int \ln x \, dx$ *requires by parts itself – making the problem harder. Logarithms always go to* u.

Question 4 Hard – Paper 2 [7 marks]

Evaluate $\displaystyle\int_1^e \ln x \, dx$.

MISTAKE ANALYSIS

By parts: $u = \ln x$, $dv = dx$, $du = \frac{1}{x} dx$, $v = x$. $\displaystyle\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C = [x \ln x - x]_1^e$. $= (e \ln e - e) - (1 \cdot \ln 1 - 1) = (e - e) - (0 - 1) = 0 + 1 = 1$. *Students who write* $e \ln e = e^2$ *confuse* $\ln e = 1$ *with* $\ln e = e$. $\ln e = 1$ *always. Students who evaluate* $[\ln x]_1^e = \ln e - \ln 1 = 1 - 0 = 1$ *without integrating have not done the question.*

Question 5 Hard – Paper 2 [7 marks]

Find the exact area under one arch of $y = \sin x$ (from $x = 0$ to $x = \pi$).

MISTAKE ANALYSIS

$\int_0^\pi \sin x\,dx = [-\cos x]_0^\pi = -\cos\pi - (-\cos 0) = -(-1) + 1 = 1 + 1 = 2$. *Students who write* $[-\cos x]_0^\pi = \cos\pi - \cos 0 = -1 - 1 = -2$ *apply the square bracket notation incorrectly:* $[f(x)]_a^b = f(b) - f(a)$, *so the lower limit is subtracted. The area is 2 square units – a result worth memorising.*

Question 6 Hard – Paper 1 [7 marks]

Find $\int \cos^2 x\,dx$.

MISTAKE ANALYSIS

Use the identity $\cos^2 x = \dfrac{1+\cos(2x)}{2}$*:* $\int \cos^2 x\,dx = \int \dfrac{1+\cos(2x)}{2}\,dx = \dfrac{x}{2} + \dfrac{\sin(2x)}{4} + C$. *Students who write* $\dfrac{\sin^2 x}{2} + C$ *integrate* $\cos x$ *to* $\sin x$ *and then square – neither step is correct.* $\cos^2 x$ *cannot be integrated by first integrating* $\cos x$. *Students who write* $\dfrac{\cos^2 x \sin x}{3} + C$ *apply the wrong formula (valid for* $\int \cos^n x \sin x\,dx$, *not* $\int \cos^2 x\,dx$*).*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

By parts twice; $2I = e^x(\sin x - \cos x)$ | $I = \dfrac{e^x(\sin x - \cos x)}{2} + C$ | **M1 A1**

Solution – Question 2

$u = x$, $v = e^x$; $xe^x - \int e^x\,dx$ | $e^x(x-1) + C$ | **M1 A1**

Solution – Question 3

$u = \ln x$, $v = x^2/2$; $\frac{x^2 \ln x}{2} - \frac{x^2}{4}$ | $\dfrac{x^2(2\ln x - 1)}{4} + C$ | **M1 A1**

Solution – Question 4

$[x\ln x - x]_1^e = (e - e) - (0 - 1)$ | 1 | **M1 A1**

Solution – Question 5

$[-\cos x]_0^\pi = -\cos\pi + \cos 0 = 1 + 1$ | 2 | **M1 A1**

Solution – Question 6

$\cos^2 x = \frac{1+\cos(2x)}{2}$; integrate | $\dfrac{x}{2} + \dfrac{\sin(2x)}{4} + C$ | **M1 A1**