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Integration

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	36 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Substitution: when the integrand contains a composite function and its derivative (or a multiple), let u equal the inner function. The dx must be converted: $du = u'\,dx$.

Area between curves: $A = \int_a^b [f(x) - g(x)]\,dx$ where $f(x) \geq g(x)$ on $[a, b]$. Find intersections first.

Rewriting: divide out or expand before integrating whenever the standard forms do not apply directly. $\frac{x+1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$.

Question 1

Hard – Paper 1 [5 marks]

Find $\int x(x^2+1)^3\,dx$.

MISTAKE ANALYSIS

Let $u = x^2 + 1$, $du = 2x\,dx$, so $x\,dx = \frac{du}{2}$. $\int u^3 \cdot \frac{du}{2} = \frac{u^4}{8} + C = \frac{(x^2+1)^4}{8} + C$. Students who write $\frac{(x^2+1)^4}{4} + C$ forget the factor $\frac{1}{2}$ from $x\,dx = \frac{du}{2}$. Alternatively, use the reverse chain rule: the inner derivative of $x^2 + 1$ is $2x$, and x is present – divide by 2.

Question 2

Hard – Paper 1 [5 marks]

Find $\int \cos x \cdot e^{\sin x}\, dx$.

MISTAKE ANALYSIS

Let $u = \sin x$, $du = \cos x\, dx$. $\int e^u\, du = e^u + C = e^{\sin x} + C$. The integrand is already in the form $f'(x) \cdot e^{f(x)}$, so the integral is directly $e^{f(x)} + C$. Students who write $e^{\sin x} \cos x + C$ include the $\cos x$ in the answer – the substitution absorbs $\cos x\, dx$ into du, so it does not remain in the result.

Question 3 Hard – Paper 1 [5 marks]

Find $\int \frac{x}{x^2+4}\, dx$.

MISTAKE ANALYSIS

Let $u = x^2 + 4$, $du = 2x\, dx$, so $x\, dx = \frac{du}{2}$. $\int \frac{1}{u} \cdot \frac{du}{2} = \frac{\ln|u|}{2} + C = \frac{\ln(x^2+4)}{2} + C$. (No absolute value needed since $x^2 + 4 > 0$ always.) Students who write $\ln(x^2+4) + C$ omit the factor $\frac{1}{2}$. Students who write $\frac{1}{2x} \ln|x^2+4| + C$ have not correctly performed the substitution.

Question 4 Hard – Paper 2 [6 marks]

Evaluate $\int_1^3 3x^2\, dx$ and interpret the result geometrically.

MISTAKE ANALYSIS

$\int_1^3 3x^2\, dx = \left[x^3\right]_1^3 = 27 - 1 = 26$. Geometrically: this is the area under the curve $y = 3x^2$ between $x = 1$ and $x = 3$. Note: $f(x) = 3x^2 = \frac{d}{dx}[x^3]$, so $\int_1^3 3x^2\, dx = x^3\Big|_1^3 = 26$ is also the net change in x^3 from $x = 1$ to $x = 3$. Students who write $[3x^3]_1^3 = 81 - 3 = 78$ integrate $3x^2$ as $3x^3$ (multiplying rather than dividing by the new power).

Question 5 Hard – Paper 2 [8 marks]

Find the area enclosed between $y = x$ and $y = x^2$.

MISTAKE ANALYSIS

Intersections: $x = x^2 \Rightarrow x(x-1) = 0 \Rightarrow x = 0$ *or* $x = 1$. *On* $[0, 1]$*:* $x \geq x^2$*, so area* $= \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. *Students who integrate without finding intersections first use the wrong limits. Students who integrate* $x^2 - x$ *(wrong order) get* $-\frac{1}{6}$*: area is always positive, so the top curve minus the bottom curve is essential.*

Question 6 Hard – Paper 1 [7 marks]

Find $\int \frac{x+1}{\sqrt{x}}\,dx$.

MISTAKE ANALYSIS

Rewrite: $\frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$. $\int (x^{1/2} + x^{-1/2})\,dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$. *Students who try to integrate* $\frac{x+1}{\sqrt{x}}$ *directly as a quotient cannot apply a standard formula. Splitting into* $x^{1/2} + x^{-1/2}$ *and applying the power rule is the key step.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$u = x^2 + 1$; $x\,dx = du/2$; $\int u^3\,du/2$ $\quad \dfrac{(x^2+1)^4}{8} + C$ **M1 A1**

Solution – Question 2

$u = \sin x$; $du = \cos x\,dx$; $\int e^u\,du$ $\quad e^{\sin x} + C$ **M1 A1**

Solution – Question 3

$u = x^2 + 4$; $x\,dx = du/2$; $\frac{1}{2}\int \frac{1}{u}\,du$ $\quad \dfrac{\ln(x^2+4)}{2} + C$ **M1 A1**

Solution – Question 4

$[x^3]_1^3 = 27 - 1$ $\quad 26;$ area under $y = 3x^2$ **M1 A1**

Solution – Question 5

Intersect $x = 0, 1$; $\int_0^1 (x - x^2)\,dx = \frac{1}{2} - \frac{1}{3}$ $\quad \dfrac{1}{6}$ **M1 A1**

Solution – Question 6

$x^{1/2} + x^{-1/2}$; power rule $\quad \dfrac{2}{3}x^{3/2} + 2x^{1/2} + C$ **M1 A1**