Integration

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	33 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Power rule: $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$.

Standard integrals: $\int e^{kx}\,dx = \dfrac{e^{kx}}{k} + C$; $\int \dfrac{1}{ax+b}\,dx = \dfrac{\ln|ax+b|}{a} + C$; $\int \sin(kx)\,dx = -\dfrac{\cos(kx)}{k} + C$.

Reverse chain rule: $\int (ax+b)^n\,dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + C$.

Constant of integration: all indefinite integrals require $+C$.

Question 1

Medium – Paper 1 [4 marks]

Find $\int (4x^3 - 3x^2 + 2x - 5)\,dx$.

MISTAKE ANALYSIS

$\int (4x^3 - 3x^2 + 2x - 5)\,dx = x^4 - x^3 + x^2 - 5x + C$. Integrate each term: $4x^3 \to x^4$, $-3x^2 \to -x^3$, $2x \to x^2$, $-5 \to -5x$. Students who write the answer without $+C$ lose the accuracy mark. Every indefinite integral requires the constant of integration.

Question 2 Medium – Paper 1 [5 marks]

Find $\int (2x+1)^4\,dx$.

MISTAKE ANALYSIS

Reverse chain rule: $\int (2x+1)^4\,dx = \dfrac{(2x+1)^5}{5\times 2} + C = \dfrac{(2x+1)^5}{10} + C$. *Students who write* $\dfrac{(2x+1)^5}{5} + C$ *forget to divide by the inner derivative 2. The denominator is* $5\times 2 = 10$, *not 5. Check by differentiating:* $\dfrac{d}{dx}\left[\dfrac{(2x+1)^5}{10}\right] = \dfrac{5(2x+1)^4\times 2}{10} = (2x+1)^4$ ✓.

Question 3 Medium – Paper 1 [5 marks]

Find $\int e^{3x}\,dx$.

MISTAKE ANALYSIS

$\int e^{3x}\,dx = \dfrac{e^{3x}}{3} + C$. *Students who write* $3e^{3x} + C$ *differentiate instead of integrating. Students who write* $e^{3x} + C$ *forget to divide by the coefficient 3. Check:* $\dfrac{d}{dx}\left[\dfrac{e^{3x}}{3}\right] = e^{3x}$ ✓.

Question 4 Hard – Paper 1 [5 marks]

Find $\int \dfrac{1}{2x+1}\,dx$.

MISTAKE ANALYSIS

$\int \dfrac{1}{2x+1}\,dx = \dfrac{\ln|2x+1|}{2} + C$. *The integral of* $\frac{1}{u}$ *is* $\ln|u|$. *With inner function* $2x+1$ *(derivative 2), divide by 2. Students who write* $\ln|2x+1| + C$ *omit the factor* $\frac{1}{2}$ *from the inner derivative. Students who write* $\dfrac{-1}{(2x+1)^2} + C$ *differentiate instead of integrating, and apply the wrong rule.*

Question 5 Hard – Paper 2 [7 marks]

Evaluate $\displaystyle\int_0^2 (x^2+1)\,dx$.

MISTAKE ANALYSIS

$\displaystyle\int_0^2 (x^2+1)\,dx = \left[\frac{x^3}{3}+x\right]_0^2 = \left(\frac{8}{3}+2\right)-(0+0) = \frac{8}{3}+\frac{6}{3} = \frac{14}{3}$. *Students who write* $\displaystyle\left[\frac{x^3}{3}+x\right]_0^2 = \frac{8}{3}+2-\frac{0}{3}+0$ *must be careful with the subtraction: the entire lower bracket* $\left(\frac{0^3}{3}+0\right)=0$ *is subtracted. Both terms at the lower limit vanish here.*

Question 6 Hard – Paper 1 [7 marks]

Find $\displaystyle\int \sin(2x)\,dx$.

MISTAKE ANALYSIS

$\displaystyle\int \sin(2x)\,dx = -\frac{\cos(2x)}{2}+C$. *Students who write* $-\cos(2x)+C$ *forget to divide by the inner derivative 2. Students who write* $+\dfrac{\cos(2x)}{2}+C$ *lose the negative sign:* $\int \sin(u)\,du = -\cos(u)$. *Check:* $\dfrac{d}{dx}\left[-\dfrac{\cos(2x)}{2}\right] = \dfrac{2\sin(2x)}{2} = \sin(2x)$ ✓.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

Power rule each term $x^4 - x^3 + x^2 - 5x + C$ **M1 A1**

Solution – Question 2

$\frac{(2x+1)^5}{5\times 2}$ $\frac{(2x+1)^5}{10} + C$ **M1 A1**

Solution – Question 3

$\int e^{kx} = e^{kx}/k$ $\frac{e^{3x}}{3} + C$ **M1 A1**

Solution – Question 4

$\int \frac{1}{ax+b}$ $\frac{\ln|ax+b|}{a}$ $= \frac{\ln|2x+1|}{2} + C$ **M1 A1**

Solution – Question 5

$\left[\frac{x^3}{3} + x\right]_0^2 = \frac{8}{3} + 2 - 0 = \frac{14}{3}$ **M1 A1**

Solution – Question 6

$\int \sin(kx)$ $-\cos(kx)/k$ $= -\frac{\cos(2x)}{2} + C$ **M1 A1**