Geometry & Trigonometry Applications

Mistake Analysis – Set III

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	39 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Angle of elevation/depression: measured from the horizontal. Draw a clear diagram before setting up the trigonometric ratio.

Coordinate geometry: midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$; distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$; gradient $= \frac{y_2-y_1}{x_2-x_1}$.

Perpendicular lines: $m_1 \times m_2 = -1$.

Full cosine rule solution: when all three sides are known, use $\cos C = \frac{a^2+b^2-c^2}{2ab}$ to find the largest angle first (it may be obtuse).

Question 1

Hard – Paper 2 [6 marks]

A person stands 40 m from the base of a building. The angle of elevation to the top of the building is 36.87. Find the height of the building.

MISTAKE ANALYSIS

$\tan(36.87) = \frac{h}{40} \Rightarrow h = 40\tan(36.87) \approx 40 \times 0.75 = 30$ *m. Students who write* $\sin(36.87) = \frac{h}{40}$ *use sine instead of tangent – the 40 m is the horizontal distance (adjacent), not the hypotenuse. Students who write* $h = \frac{40}{\tan(36.87)} \approx 53.3$ *m invert the ratio.*

Question 2 Hard – Paper 2 [6 marks]

A box has dimensions 6 cm by 4 cm by 5 cm. Find the angle that the space diagonal makes with the base of the box.

MISTAKE ANALYSIS

Base diagonal $= \sqrt{6^2+4^2} = \sqrt{52} = 2\sqrt{13} \approx 7.211$ *cm. Angle with base* $= \arctan\left(\dfrac{5}{2\sqrt{13}}\right) \approx \arctan(0.6934) \approx 34.7$. *Students who use the space diagonal as the hypotenuse for a right triangle with vertical height 5: correct setup –* $\sin\theta = \frac{5}{\sqrt{77}}$ *gives the same angle. But the two-step method (base diagonal first, then trigonometry) is clearest. Students who write angle* $= \arctan\left(\frac{5}{6}\right)$ *use only one base dimension, ignoring the width.*

Question 3 Hard – Paper 1 [6 marks]

Points $A(2,3)$ and $B(8,11)$. Find the midpoint M of AB, the distance AB, and the gradient of AB.

MISTAKE ANALYSIS

$M = \left(\dfrac{2+8}{2}, \dfrac{3+11}{2}\right) = (5,7)$. $AB = \sqrt{(8-2)^2+(11-3)^2} = \sqrt{36+64} = \sqrt{100} = 10$. *Gradient* $= \dfrac{11-3}{8-2} = \dfrac{8}{6} = \dfrac{4}{3}$. *Students who write gradient* $= \frac{8-2}{11-3} = \frac{6}{8} = \frac{3}{4}$ *invert the formula: gradient* $= \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$, *not* $\frac{\Delta x}{\Delta y}$.

Question 4 Hard – Paper 1 [6 marks]

A line passes through the points $(1,2)$ and $(5,10)$. Find the equation of the line perpendicular to this, passing through the midpoint of the two given points.

MISTAKE ANALYSIS

Gradient of $AB = \dfrac{10-2}{5-1} = \dfrac{8}{4} = 2$. *Perpendicular gradient* $= -\dfrac{1}{2}$. *Midpoint* $= (3,6)$. *Perpendicular:* $y-6 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{x}{2} + \frac{15}{2}$. *Students who use the same gradient (2) for the perpendicular instead of* $-\frac{1}{2}$. *Perpendicular lines have gradients whose product is* -1.

Question 5 Hard – Paper 2 [7 marks]

A triangle has sides $a = 5$ cm, $b = 7$ cm, $c = 9$ cm. Find the largest angle.

MISTAKE ANALYSIS

The largest angle is opposite the longest side: $c = 9$, so find C. $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{25 + 49 - 81}{70} = \dfrac{-7}{70} = -0.1$. $C = \arccos(-0.1) \approx 95.7$. *The negative cosine confirms C is obtuse. Students who write* $\cos C = \frac{a^2+b^2+c^2}{2ab}$ *use a plus sign – the formula always has minus in the numerator. Students who stop at* $\cos C = -0.1$ *without finding the angle lose the accuracy mark.*

Question 6 Hard – Paper 2 [8 marks]

Using the same triangle from Question 5 ($a = 5$, $b = 7$, $c = 9$), find the area of the triangle.

MISTAKE ANALYSIS

Using $C \approx 95.74$: Area $= \dfrac{1}{2}ab\sin C = \dfrac{1}{2}(5)(7)\sin(95.74) \approx 17.5 \times 0.9950 \approx 17.41$ *cm². Carry the exact C from Q5. Alternatively: Heron's formula with* $s = \frac{5+7+9}{2} = \frac{21}{2}$*:* $A = \sqrt{\frac{21}{2} \cdot \frac{11}{2} \cdot \frac{7}{2} \cdot \frac{3}{2}} = \sqrt{\frac{21\times11\times7\times3}{16}} = \sqrt{\frac{4851}{16}} \approx 17.41$ *cm². Students who use Area* $= \frac{1}{2} \times b \times h$ *without knowing h cannot proceed –* $\frac{1}{2}ab\sin C$ *is the correct approach once the angle is known.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$h = 30 \text{ m}$ — **M1 A1**

$40\tan(36.87)$

Solution – Question 2

Base diag ≈ 34.7 — **M1 A1**

$= \sqrt{52}$; angle $=$ $\arctan(5/\sqrt{52})$

Solution – Question 3

$M =$ — **M1 A1**

$(5, 7)$; $AB =$ 10; grad $= 4/3$

Solution – Question 4

Grad $y = -x/2 + 15/2$ — **M1 A1**

$= 2$; perp grad $= -1/2$; midpt $(3, 6)$

Solution – Question 5

$\cos C = (25 + C \approx 95.7$ — **M1 A1**

$49 - 81)/70 =$ -0.1

Solution – Question 6

$\frac{1}{2}(5)(7)\sin(95.7) \approx 17.4 \text{ cm}^2$ — **M1 A1**