Geometry & Trigonometry Applications

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	36 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Cosine rule (angle form): $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$.

Sine rule – ambiguous case: when finding an angle using $\sin B = \dfrac{b \sin A}{a}$, there may be two solutions: B and $180 - B$. Check both against the angle sum condition.

3D geometry: space diagonal of a box: $d = \sqrt{l^2 + w^2 + h^2}$. To find an angle with the base, use the base diagonal and the height.

Segment area: Area of segment = Area of sector − Area of triangle $= \dfrac{1}{2} r^2(\theta - \sin\theta)$.

Question 1

Medium – Paper 1 [5 marks]

In triangle ABC, $a = 6$ cm, $b = 8$ cm, $c = 10$ cm. Find angle C.

MISTAKE ANALYSIS

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{36 + 64 - 100}{96} = \dfrac{0}{96} = 0 \Rightarrow C = 90$. *Students who immediately recognise* 6-8-10 *as a scaled* 3-4-5 *triangle should still show the cosine rule working for full marks. Students who write* $\cos C = \frac{c^2}{a^2+b^2} = \frac{100}{100} = 1 \Rightarrow C = 0$ *have the formula inverted.*

Question 2

Hard – Paper 2 [6 marks]

In triangle ABC, $a = 7$ cm, $b = 9$ cm, $A = 50$. Find all possible values of angle B.

MISTAKE ANALYSIS

$\sin B = \dfrac{b\sin A}{a} = \dfrac{9\sin 50}{7} = \dfrac{9 \times 0.7660}{7} \approx 0.9849$. $B \approx 80.0$ *or* $B \approx 180 - 80.0 = 100.0$. *Both values are valid since* $50 + 80 = 130 < 180$ *and* $50 + 100 = 150 < 180$. *The ambiguous case arises when using the sine rule to find an angle. Both solutions must be checked; both are valid here. Students who report only* $B \approx 80$ *miss the obtuse solution.*

Question 3 Hard – Paper 2 [5 marks]

A box has dimensions 4 cm by 5 cm by 6 cm. Find the length of the space diagonal.

MISTAKE ANALYSIS

$d = \sqrt{4^2 + 5^2 + 6^2} = \sqrt{16 + 25 + 36} = \sqrt{77} \approx 8.77$ *cm. Students who apply Pythagoras in 2D only:* $d = \sqrt{4^2 + 5^2} = \sqrt{41}$, *then stop – this gives the face diagonal, not the space diagonal. The space diagonal requires all three dimensions.*

Question 4 Hard – Paper 2 [6 marks]

A ship sails 50 km due North, then 30 km due East. Find the total distance from the starting point and the bearing of the ship from the starting point.

MISTAKE ANALYSIS

Distance $= \sqrt{50^2 + 30^2} = \sqrt{3400} = 10\sqrt{34} \approx 58.3$ *km. Bearing:* $\tan\theta = \dfrac{30}{50} = 0.6 \Rightarrow \theta \approx 31.0$. *Bearing from start* $= N31.0E = 031$. *Students who compute* $\arctan\frac{50}{30}$ *invert the fraction – North is the vertical direction (adjacent), East is horizontal (opposite). The bearing angle is measured from North, so* $\tan\theta = \frac{East}{North}$.

Question 5 Hard – Paper 1 [6 marks]

A sector of a circle has radius 8 cm and arc length 12 cm. Find the central angle in radians and the area of the sector.

MISTAKE ANALYSIS

$\theta = \dfrac{arc}{r} = \dfrac{12}{8} = 1.5$ *radians.* *Area* $= \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(64)(1.5) = 48$ cm^2. *Students who convert 1.5 rad to degrees* (≈ 85.9) *before finding the area: valid but unnecessary. The radian formula works directly. Students who compute area* $= r\theta = 8 \times 1.5 = 12$ *confuse arc length with sector area.*

Question 6 Hard – Paper 1 [8 marks]

A sector of a circle has radius 10 cm and central angle $\dfrac{\pi}{4}$ radians. Find the area of the minor segment (the region between the chord and the arc). Give an exact answer.

MISTAKE ANALYSIS

Sector area $= \dfrac{1}{2}(100)\dfrac{\pi}{4} = \dfrac{25\pi}{2}$. *Triangle area* $= \dfrac{1}{2}(10)(10)\sin\dfrac{\pi}{4} = 50 \times \dfrac{\sqrt{2}}{2} = 25\sqrt{2}$. *Segment area* $= \dfrac{25\pi}{2} - 25\sqrt{2} = 25\left(\dfrac{\pi}{2} - \sqrt{2}\right)$ cm^2. *Students who write segment = sector only, omitting to subtract the triangle. Students who use* $\sin\frac{\pi}{4} = \frac{1}{2}$ *instead of* $\frac{\sqrt{2}}{2}$*:* $\sin\frac{\pi}{4} = \sin 45 = \frac{\sqrt{2}}{2}$*, not* $\frac{1}{2}$. $\sin\frac{\pi}{6} = \frac{1}{2}$.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$\cos C = (36 + 64 - 100)/96 = 0$ $C = 90$ **M1 A1**

Solution – Question 2

$\sin B = 9\sin 50/7 \approx 0.9849$; both solutions valid $B \approx 80.0$ or $B \approx 100.0$ **M1 A1 R1**

Solution – Question 3

$\sqrt{16+25+36}$ $\sqrt{77} \approx 8.77$ cm **M1 A1**

Solution – Question 4

$\sqrt{50^2+30^2} \approx 58.3$ km; $10\sqrt{34}$; $\theta = \arctan(30/50) \approx 31.0$ 031 **M1 A1**

Solution – Question 5

$\theta = 12/8 = 1.5$ rad; Area $= \frac{1}{2}(64)(1.5)$ 48 cm^2 **M1 A1**

Solution – Question 6

Sector $= 25\left(\frac{\pi}{2} - \sqrt{2}\right)$ cm^2 **M1 A1**

$25\pi/2$; Triangle $= 25\sqrt{2}$; Segment $=$