

Geometry & Trigonometry Applications

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 3 – Geometry & Trigonometry
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	34 total. M1 method · A1 accuracy · R1 reasoning.

BEFORE YOU BEGIN

Right triangles: $a^2 + b^2 = c^2$ (Pythagoras); $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$.

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$.

Area: $\text{Area} = \frac{1}{2}ab \sin C$.

Radians: arc length = $r\theta$; sector area = $\frac{1}{2}r^2\theta$.

Question 1

Medium – Paper 1

[4 marks]

A right triangle has legs of length 5 cm and 12 cm. Find the length of the hypotenuse.

MISTAKE ANALYSIS

$c = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ cm. Students who write $c = 5 + 12 = 17$ add the sides instead of applying Pythagoras. Students who write $c = \sqrt{5^2 + 12^2} = \sqrt{17^2} = 17$ compute $5 + 12 = 17$ inside the root – $\sqrt{169} \neq \sqrt{17^2}$.

Question 2

Medium – Paper 2

[5 marks]

In a right triangle, the side opposite to angle θ has length 7 cm and the hypotenuse is 10 cm. Find θ , correct to 2 decimal places.

MISTAKE ANALYSIS

$\sin \theta = \frac{7}{10} = 0.7$. $\theta = \arcsin(0.7) \approx 44.43$. *Students who write $\tan \theta = \frac{7}{10}$ use the tangent ratio instead of sine. The tangent uses opposite over adjacent, not opposite over hypotenuse. Students who write $\theta = \frac{7}{10} = 0.7$ confuse the sine value with the angle.*

Question 3

Hard – Paper 2

[6 marks]

In triangle ABC , $a = 8$ cm, $A = 40$, $B = 65$. Find the length of side b .

MISTAKE ANALYSIS

$\frac{b}{\sin B} = \frac{a}{\sin A}$: $b = \frac{8 \sin 65}{\sin 40} \approx \frac{8 \times 0.9063}{0.6428} \approx 11.28$ cm. *Students who write $\frac{b}{\sin A} = \frac{a}{\sin B}$ invert the sine rule – each side is divided by the sine of the opposite angle. Students who find $C = 180 - 40 - 65 = 75$ first to check: C is not needed for this calculation but verifies the triangle is valid.*

Question 4

Hard – Paper 1

[6 marks]

In triangle ABC , $a = 5$ cm, $b = 7$ cm, $C = 60$. Find the length of side c .

MISTAKE ANALYSIS

$c^2 = a^2 + b^2 - 2ab \cos C = 25 + 49 - 2(5)(7) \cos 60 = 74 - 70 \times \frac{1}{2} = 74 - 35 = 39$. $c = \sqrt{39} \approx 6.24$ cm. *Students who write $c^2 = a^2 + b^2 + 2ab \cos C$ use the wrong sign – it is minus, not plus. Students who use $\cos 60 = \frac{\sqrt{3}}{2}$ confuse $\cos 60$ with $\sin 60$: $\cos 60 = \frac{1}{2}$.*

Question 5

Hard – Paper 2

[6 marks]

In triangle ABC , $a = 6$ cm, $b = 9$ cm, $C = 50$. Find the area of the triangle.

MISTAKE ANALYSIS

$Area = \frac{1}{2}ab \sin C = \frac{1}{2}(6)(9) \sin 50 = 27 \sin 50 \approx 27 \times 0.7660 \approx 20.68 \text{ cm}^2$. *Students who write Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(6)(9) = 27$ omit $\sin C$ and treat the two sides as base and height. The formula $\frac{1}{2}ab \sin C$ is needed when the height is not given directly.*

Question 6

Hard – Paper 1

[7 marks]

A sector of a circle has radius 8 cm and central angle $\frac{\pi}{3}$ radians. Find the arc length and the area of the sector. Give exact answers.

MISTAKE ANALYSIS

$Arc \text{ length} = r\theta = 8 \times \frac{\pi}{3} = \frac{8\pi}{3} \text{ cm}$. $Area = \frac{1}{2}r^2\theta = \frac{1}{2}(64)\frac{\pi}{3} = \frac{32\pi}{3} \text{ cm}^2$. *Students who use the degree formula $\frac{\theta}{360} \times 2\pi r$ after converting $\frac{\pi}{3}$ to 60: valid but longer. The radian formulae $r\theta$ and $\frac{1}{2}r^2\theta$ are direct and exact. Students who write $area = r^2\theta = \frac{64\pi}{3}$ omit the factor $\frac{1}{2}$.*

WORKED SOLUTIONS – SET I – GEOMETRY & TRIGONOMETRY APPLICATIONS

M1 method · A1 accuracy · R1 reasoning

Solution – Question 1

$$\frac{\sqrt{25 + 144}}{\sqrt{169}} = 13 \text{ cm} \quad \text{M1 A1}$$

Solution – Question 2

$$\begin{aligned} \sin \theta &= 44.43 & \text{M1 A1} \\ 7/10; \theta &= \\ \arcsin(0.7) & \end{aligned}$$

Solution – Question 3

$$\begin{aligned} b &= \approx 11.28 \text{ cm} & \text{M1 A1} \\ 8 \sin 65 / \sin 40 & \end{aligned}$$

Solution – Question 4

$$\begin{aligned} c^2 &= \sqrt{39} \approx 6.24 \text{ cm} & \text{M1 A1} \\ 74 - 70 \cos 60 &= \\ 74 - 35 = 39 & \end{aligned}$$

Solution – Question 5

$$\begin{aligned} \frac{1}{2}(6)(9) \sin 50 &= \approx 20.68 \text{ cm}^2 & \text{M1 A1} \\ 27 \sin 50 & \end{aligned}$$

Solution – Question 6

$$\begin{aligned} \text{Arc} &= \text{Arc} = \frac{8\pi}{3} \text{ cm}; \text{Area} = \frac{32\pi}{3} \text{ cm}^2 & \text{M1 A1} \\ 8\pi/3; \text{Area} & \\ = \frac{1}{2}(64)(\pi/3) & \end{aligned}$$
