Functions

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 2 – Functions: Quadratics, Graphs & Transformations
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	35 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Vertex form: $f(x) = a(x-h)^2 + k$ has vertex (h, k). Complete the square to convert from $ax^2 + bx + c$.

Discriminant: $\Delta = b^2 - 4ac$. Equal roots when $\Delta = 0$; two real roots when $\Delta > 0$; none when $\Delta < 0$.

Roots and factors: if p and q are roots, $f(x) = a(x-p)(x-q)$.

Reciprocal graphs: $y = \dfrac{1}{x-h} + k$ has vertical asymptote $x = h$ and horizontal asymptote $y = k$.

Question 1

Medium – Paper 1 [5 marks]

Express $f(x) = 2x^2 - 12x + 19$ in the form $a(x-h)^2 + k$, and state the coordinates of the vertex.

MISTAKE ANALYSIS

Factor the coefficient of x^2 from the first two terms: $f(x) = 2(x^2 - 6x) + 19$. Complete the square inside: $x^2 - 6x = (x-3)^2 - 9$. So $f(x) = 2[(x-3)^2 - 9] + 19 = 2(x-3)^2 - 18 + 19 = 2(x-3)^2 + 1$. Vertex: $(3, 1)$. Students who forget to multiply the -9 by the factored-out 2 write $f(x) = 2(x-3)^2 - 9 + 19 = 2(x-3)^2 + 10$ – wrong. The -9 is inside the bracket, so it must be multiplied by 2 when removed: $2 \times (-9) = -18$.

Question 2

Medium – Paper 1 [5 marks]

Find the values of k for which the equation $x^2+kx+9=0$ has two equal real roots.

MISTAKE ANALYSIS

Equal roots require discriminant $=0$: $\Delta = k^2-4(1)(9) = k^2-36 = 0 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$. Students who give only $k=6$ miss $k=-6$. The equation $k^2=36$ has two solutions. Students who write $\Delta = k^2 - 4\times 9 = k^2-36$ but then solve $k^2-36>0$ answer the wrong condition. "Equal roots" means $\Delta = 0$ exactly, not $\Delta > 0$.

Question 3 Hard – Paper 1 [6 marks]

A quadratic function has roots at $x=-2$ and $x=5$, and its graph passes through the point $(0,-20)$. Find the function in the form $f(x)=ax^2+bx+c$.

MISTAKE ANALYSIS

From the roots: $f(x)=a(x+2)(x-5)$. Use $(0,-20)$: $f(0)=a(2)(-5)=-10a=-20 \Rightarrow a=2$. $f(x)=2(x+2)(x-5)=2(x^2-3x-10)=2x^2-6x-20$. Students who write $f(x)=(x+2)(x-5)$ without the leading coefficient a cannot match the point $(0,-20)$: that form gives $f(0)=-10\neq -20$. The factor a is essential; find it using the given point.

Question 4 Hard – Paper 1 [6 marks]

Determine the number of points of intersection of the line $y=2x-3$ and the parabola $y=x^2-2x+1$, and find any point(s) of intersection.

MISTAKE ANALYSIS

Set equal: $x^2-2x+1=2x-3 \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$ (repeated). One point of intersection: at $x=2$, $y=2(2)-3=1$. The line is tangent to the parabola at $(2,1)$. The discriminant of x^2-4x+4 is $16-16=0$, confirming exactly one intersection (tangency). Students who find $(x-2)^2=0$ and report two solutions $x=2, x=2$ should recognise this as a single (repeated) point, meaning the line touches the parabola once.

Question 5 Hard – Paper 1 [6 marks]

The function $f(x) = \frac{1}{x-2} + 3$ is a transformation of $y = \frac{1}{x}$.

(a) State the equations of the two asymptotes.

(b) Find the coordinates of the point where the graph crosses the x-axis.

MISTAKE ANALYSIS

(a) Vertical asymptote: $x = 2$ *(denominator zero). Horizontal asymptote:* $y = 3$ *(the* $+3$ *shift). (b)* x*-axis: set* $f(x) = 0$*:* $\frac{1}{x-2} + 3 = 0 \Rightarrow \frac{1}{x-2} = -3 \Rightarrow x - 2 = -\frac{1}{3} \Rightarrow x = \frac{5}{3}$*. Crosses at* $\left(\frac{5}{3}, 0\right)$*. Students who state the horizontal asymptote as* $y = 0$ *forget the vertical shift of* $+3$*. The base function* $y = 1/x$ *has asymptote* $y = 0$*, but adding 3 shifts it to* $y = 3$*.*

Question 6 Medium – Paper 1 [7 marks]

Consider $f(x) = |x - 3| + 2$.

(a) State the coordinates of the vertex and the minimum value.

(b) Sketch the graph, and state the range.

(c) Solve $f(x) = 6$.

MISTAKE ANALYSIS

(a) Vertex at $(3, 2)$*; minimum value* 2 *(absolute value is never negative, minimum 0, plus 2). (b) V-shape opening upward, vertex* $(3, 2)$*. Range:* $f(x) \geq 2$*, i.e.* $[2, \infty)$*. (c)* $|x-3| + 2 = 6 \Rightarrow |x - 3| = 4 \Rightarrow x - 3 = 4$ *or* $x - 3 = -4 \Rightarrow x = 7$ *or* $x = -1$*. Students who state the range as* $f(x) \geq 0$ *forget the vertical shift. The minimum of* $|x-3|$ *is 0, so the minimum of* $|x-3|+2$ *is 2. For part (c), students who solve only* $x - 3 = 4$ *miss the second solution* $x = -1$*.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$2(x^2-6x)+19 = 2[(x-3)^2-9]+19$	$2(x-3)^2+1$	**M1 A1**
Vertex	$(3,1)$	**A1**

Solution – Question 2

$\Delta = k^2 - 36 = 0$	$k = \pm 6$	**M1 A1**

Solution – Question 3

$f(x) = a(x+2)(x-5)$; $f(0) = -10a = -20 \Rightarrow a = 2$		**M1 A1**
$2(x+2)(x-5)$	$2x^2 - 6x - 20$	**A1**

Solution – Question 4

$x^2-4x+4 = (x-2)^2 = 0$; $\Delta = 0$ (tangent)	one point	**M1 A1**
At $x = 2$, $y = 1$	$(2,1)$	**A1**

Solution – Question 5

(a) Asymptotes	$x = 2,\ y = 3$	**A1**
(b) $\frac{1}{x-2} = -3 \Rightarrow x - 2 = -\frac{1}{3}$	$\left(\frac{5}{3}, 0\right)$	**M1 A1**

Solution – Question 6

(a) Vertex $(3, 2)$; min value 2		**A1**
(b) V-shape; range	$[2, \infty)$	**A1**
(c) $\lvert x - 3 \rvert = 4$	$x = 7$ or $x = -1$	**M1 A1**

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