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# Financial Mathematics

*Mistake Analysis – Set III*

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| **Course** | IB Mathematics: Analysis & Approaches SL |
| **Topic** | Topic 1 – Number & Algebra: Financial Mathematics |
| **Level** | Hard (Paper 2 – calculator permitted) |
| **Questions** | 6 |
| **Marks** | 37 total. **M1** method · **A1** accuracy · **R1** reasoning. |

## BEFORE YOU BEGIN

**Two-equation system:** when given two data points, divide one equation by the other to eliminate $P$ and solve for $r$, then substitute back.

**When two models are equal:** set them equal and solve – isolate the exponential ratio, then take logarithms.

**Depreciation below a threshold:** set up an inequality $(1-r)^n <$ threshold, take logarithms, and round up to the next complete period.

**Continuous compounding:** $A = P\,e^{rt}$ where $r$ is the continuous rate and $t$ is time in years.

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### Question 1

Hard – Paper 2 [6 marks]

$P is invested at $r$% per annum compound interest. After 10 years the amount is \$12 100. After 20 years the amount is \$14 641. Find $P$ and $r$.

**MISTAKE ANALYSIS**

*Equations:* $P(1+r/100)^{10} = 12100$ *…(1) and* $P(1+r/100)^{20} = 14641$ *…(2). Divide (2) by (1):* $(1+r/100)^{10} = 14641/12100 = 1.21$. $1+r/100 = 1.21^{1/10} \Rightarrow r/100 = 1.21^{0.1}-1 \approx 0.01925 \Rightarrow r \approx 1.92\%$. *From (1):* $P = 12100/(1.21^{1/10})^{10} = 12100/1.21 = \$10\,000$. *Verify:* $10000 \times (1.01925)^{10} \approx 12100\,\checkmark$; $10000 \times (1.01925)^{20} \approx 14641\,\checkmark$. *Students who subtract equation (1) from (2) cannot eliminate* $P$. *Division is the key step – it removes* $P$ *and leaves a single equation in* $r$.

**Question 2** Hard – Paper 2 [5 marks]

A teacher's annual salary is currently \$45 000. The salary increases by 3% each year. Find the salary after 8 years, correct to 2 decimal places.

**MISTAKE ANALYSIS**

$S = 45000 \times (1.03)^8 = 45000 \times 1.26677\ldots = \$57\,004.65$. *Salary growth follows the same compound model as investment growth – each year the increase is 3% of the current salary, not 3% of the original. Students who add* $8 \times 3\% \times 45000 = \$10\,800$ *and state the final salary as* $\$55\,800$ *use straight-line (simple) growth. Over 8 years, the compound model gives* $\$2\,204.65$ *more because later years grow on a larger base.*

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**Question 3** Hard – Paper 2 [7 marks]

Account A holds \$3 000 at 4% per annum compound interest. Account B holds \$2 500 at 6% per annum compound interest. Both accounts start at the same time. Find the time, in years to 3 significant figures, when the two accounts hold equal amounts.

**MISTAKE ANALYSIS**

*Set equal:* $3000 \times (1.04)^n = 2500 \times (1.06)^n$. $\dfrac{(1.04)^n}{(1.06)^n} = \dfrac{2500}{3000} = \dfrac{5}{6}$. $\left(\dfrac{1.04}{1.06}\right)^n = \dfrac{5}{6} \Rightarrow n\ln\left(\dfrac{1.04}{1.06}\right) = \ln\left(\dfrac{5}{6}\right) \Rightarrow n = \dfrac{\ln(5/6)}{\ln(1.04/1.06)} \approx 9.57$ *years. Students who write* $3000 \times 0.04 \times n = 2500 \times 0.06 \times n$ *equate the annual interest amounts, not the total balances – this gives* $120n = 150n$*, which has no solution. Set the full compound balances equal.*

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**Question 4** Hard – Paper 2 [5 marks]

A machine is purchased for \$80 000 and depreciates at 15% per year. Find the minimum number of complete years for the machine's value to fall below half its purchase price.

**MISTAKE ANALYSIS**

$80000 \times (0.85)^n < 40000 \Rightarrow (0.85)^n < 0.5 \Rightarrow n\ln(0.85) < \ln(0.5)$. *Since* $\ln(0.85) < 0$*, dividing flips the inequality:* $n > \dfrac{\ln(0.5)}{\ln(0.85)} \approx \dfrac{-0.6931}{-0.1625} \approx 4.27$. *Minimum complete years:* $n = 5$. *Verify:* $(0.85)^5 \approx 0.4437 < 0.5\,\checkmark$; $(0.85)^4 \approx 0.5220 > 0.5\,\checkmark$. *Students who forget to flip the inequality when dividing by* $\ln(0.85) < 0$

*get $n < 4.27$, giving a wrong answer of $n = 4$. Dividing by a negative number reverses the direction of an inequality.*

**Question 5** Hard – Paper 2 [7 marks]

A couple plan to buy a car for \$50 000 in 10 years. How much must they invest today at 4.5% per annum compound interest to have exactly this amount? Give your answer correct to 2 decimal places.

**MISTAKE ANALYSIS**

$P \times (1.045)^{10} = 50000 \Rightarrow P = \frac{50000}{(1.045)^{10}} = \frac{50000}{1.55297\ldots} = \$32\,196.38$. *Present value: the amount needed today to reach a target future value. Divide the target by the growth factor, not multiply. Students who compute* $50000 \times (1.045)^{10} = \$77\,648$ *find the future value of \$50 000 invested today – the reverse of what is asked. Present value divides by the growth factor; future value multiplies.*

**Question 6** Hard – Paper 2 [7 marks]

\$10 000 is invested for 5 years.
Plan A: 5% per annum, compounded annually.
Plan B: 5.1% per annum, compounded continuously ($A = Pe^{rt}$).
Find the value of each investment after 5 years and determine which plan is better, giving a reason.

**MISTAKE ANALYSIS**

*Plan A:* $A = 10000 \times (1.05)^5 = 10000 \times 1.27628\ldots = \$12\,762.82$. *Plan B:* $A = 10000 \times e^{0.051\times 5} = 10000 \times e^{0.255} = 10000 \times 1.29046\ldots = \$12\,904.62$. *Plan B gives* $\$12\,904.62 - \$12\,762.82 = \$141.80$ *more. Plan B is better despite appearing to have a similar rate to Plan A, because continuous compounding applies interest at every instant, making the effective annual rate* $e^{0.051} - 1 \approx 5.23\%$*, which exceeds Plan A's* 5%*. Students who compare* 5% *and* 5.1% *directly and state "Plan B is slightly better" without computing – this approach is correct in direction but does not quantify the difference. Calculate both amounts.*

**M1** method · **A1** accuracy · **R1** reasoning

### Solution – Question 1

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|---|---|---|
| Divide: $(1+r/100)^{10} = 14641/12100 = 1.21$ | $r \approx 1.92\%$ | **M1 A1** |
| $P = 12100/1.21$ | \$10 000 | **A1** |

### Solution – Question 2

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|---|---|---|
| $S = 45000 \times (1.03)^8$ | \$57 004.65 | **M1 A1** |

### Solution – Question 3

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|---|---|---|
| $(1.04/1.06)^n = 5/6 \Rightarrow n = \ln(5/6)/\ln(1.04/1.06)$ | $n \approx 9.57$ years | **M1 A1** |

### Solution – Question 4

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|---|---|---|
| $(0.85)^n < 0.5 \Rightarrow n > \ln(0.5)/\ln(0.85) \approx 4.27$; inequality flips (dividing by negative) | $n = 5$ years | **M1 A1 R1** |

### Solution – Question 5

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|---|---|---|
| $P = 50000/(1.045)^{10}$ | \$32 196.38 | **M1 A1** |

### Solution – Question 6

A: $10000 \times (1.05)^5 = \$12\,762.82$; B: $10000 \times e^{0.255} = \$12\,904.62$ **M1 A1**

Plan B better by \$141.80; EAR of B $= e^{0.051} - 1 \approx 5.23\% > 5\%$ **R1**

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