THE SANCTUARY OF LEARNING – ABDULWADOOD.ORG

Financial Mathematics

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 1 – Number & Algebra: Financial Mathematics
Level	Medium → Hard (Paper 2 – calculator permitted)
Questions	6
Marks	35 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Present value: $P = \dfrac{A}{\left(1+\frac{r}{100}\right)^n}$. The amount needed now to reach a target A in n years.

Effective annual rate: $r_{\text{eff}} = \left(1 + \dfrac{r_{\text{nom}}}{100k}\right)^k - 1$ where k is the number of compounding periods per year.

Finding n: isolate the exponential, then use logarithms. Round up if the question asks for a minimum number of complete periods.

Question 1

Medium – Paper 2 [5 marks]

How much must be invested now at 4% per annum compound interest to accumulate \$10 000 in 5 years? Give your answer correct to 2 decimal places.

MISTAKE ANALYSIS

$P \times (1.04)^5 = 10000 \Rightarrow P = \dfrac{10000}{(1.04)^5} = \dfrac{10000}{1.21665\ldots} = \$8219.27$. *This is the present value (PV) – the amount today worth \$10 000 in 5 years at 4%. Students who multiply instead of divide:* $P = 10000 \times (1.04)^5 = \$12\,166.53$ *– this computes the future value of \$10 000, not the present value. To find the present value, divide the future amount by the growth factor.*

Question 2

Medium – Paper 2 [5 marks]

An item costs \$120 today. If inflation runs at 3% per annum, find the cost of the item in 8 years, giving your answer correct to 2 decimal places.

MISTAKE ANALYSIS

$C = 120 \times (1.03)^8 = 120 \times 1.26677\ldots = \$152.01$. *Inflation increases prices like compound interest – each year the price grows by 3% of its current value, not 3% of the original price. Students who use simple inflation:* $120 + 8 \times 0.03 \times 120 = 120 + 28.8 = \$148.80$ *– this applies 3% of the original price each year. The correct model is compound:* $120 \times (1.03)^8$.

Question 3

Hard – Paper 2 [6 marks]

A bank offers a nominal annual rate of 8% compounded quarterly.

(a) Find the effective annual rate, correct to 4 decimal places.

(b) Compare this with a different account offering 8.1% compounded annually.

MISTAKE ANALYSIS

(a) $r_{\text{eff}} = \left(1 + \frac{0.08}{4}\right)^4 - 1 = (1.02)^4 - 1 = 1.08243\ldots - 1 = 0.08243 = 8.2432\%$. *(b) The first account's effective rate (8.2432%) is higher than the second account (8.1%), so the first account is better despite having the same nominal rate. Students who state "both accounts offer 8%" are comparing nominal rates. When compounding is more frequent, the effective rate exceeds the nominal rate. The effective annual rate is the only fair basis for comparison.*

Question 4

Medium – Paper 2 [5 marks]

\$15 000 is borrowed at 7% per annum compound interest. If no repayments are made, find the amount owed after 3 years.

MISTAKE ANALYSIS

$A = 15000 \times (1.07)^3 = 15000 \times 1.22504\ldots = \$18\,375.65$. *Students who calculate interest only:* $15000 \times 0.07 \times 3 = \$3\,150$, *giving total* $= \$18\,150$ *– simple interest. Compound interest charges interest on the growing balance: after year 1 the balance is* \$16 050; *year 2 interest is* 7% *of* \$16 050, *not* 7% *of* \$15 000.

Question 5 Hard – Paper 2 [7 marks]

An investment grows at 6% per annum compound interest. Find the minimum number of complete years for the investment to triple in value.

MISTAKE ANALYSIS

$(1.06)^n > 3 \Rightarrow n > \dfrac{\ln 3}{\ln 1.06} \approx \dfrac{1.0986}{0.05827} \approx 18.85$. *Minimum complete years:* $n = 19$. *Verify:* $(1.06)^{19} \approx 3.026 > 3\checkmark$; $(1.06)^{18} \approx 2.854 < 3\checkmark$. *Students who use* $n = 3/0.06 = 50$ *(dividing the multiplier by the rate) apply a simple interest formula. Logarithms are essential for compound growth problems where the time is unknown.*

Question 6 Hard – Paper 2 [7 marks]

Account A: \$6 000 invested at 5.5% per annum compounded annually for 4 years.
Account B: \$6 000 invested at 5.4% per annum compounded monthly for 4 years.
Determine which account gives the greater return and by how much, correct to 2 decimal places.

MISTAKE ANALYSIS

Account A: $6000 \times (1.055)^4 = 6000 \times 1.238825 = \$7432.95$. *Account B:* $6000 \times \left(1 + \dfrac{0.054}{12}\right)^{48} = 6000 \times (1.0045)^{48} = 6000 \times 1.240501 = \$7443.01$. *Account B gives the greater return by* $\$7443.01 - \$7432.95 = \$10.06$. *The nominal rate of B* (5.4%) *is lower than A* (5.5%)*, but monthly compounding gives B an effective annual rate of* $(1.0045)^{12} - 1 \approx 5.54\%$*, which exceeds A's* 5.5%*. This is why B wins despite the lower nominal rate. Students who compare only the nominal rates* $(5.5\%$ *vs* $5.4\%)$ *conclude A is better – this ignores compounding frequency. Always compute both final amounts or compare effective annual rates.*

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$P = 10000 \div (1.04)^5$ | $8219.27 | **M1 A1**

Solution – Question 2

$C = 120 \times (1.03)^8$ | $152.01 | **M1 A1**

Solution – Question 3

(a) $(1.02)^4 - 1 = 0.08243$ | 8.2432% | **M1 A1**

(b) Effective rate $8.2432\% > 8.1\%$: Account 1 is better | **R1**

Solution – Question 4

$A = 15000 \times (1.07)^3$ | $18 375.65 | **M1 A1**

Solution – Question 5

$(1.06)^n > 3 \Rightarrow n > \ln 3/\ln 1.06 \approx 18.85$; round up | $n = 19$ years | **M1 A1 R1**

Solution – Question 6

A: $6000 \times (1.055)^4 = \$7432.95$; B: $6000 \times (1.0045)^{48} = \$7443.01$	**M1 A1**
B is greater by \$10.06; EAR of B $\approx 5.536\% > 5.5\%$: more frequent compounding	**R1**

IB Mathematics AA SL · Mistake Analysis · Financial Mathematics Set II · Version 1 · The Sanctuary of Learning · abdulwadood.org