

Financial Mathematics

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 1 – Number & Algebra: Financial Mathematics
Level	Medium → Hard (Paper 2 style – calculator permitted)
Questions	6
Marks	34 total. M1 method · A1 accuracy · R1 reasoning.

BEFORE YOU BEGIN

Compound interest: $A = P\left(1 + \frac{r}{100}\right)^n$ where P is principal, $r\%$ is the annual rate, n is the number of years.

Compounded k times per year: $A = P\left(1 + \frac{r}{100k}\right)^{kn}$.

Depreciation: $V = P\left(1 - \frac{r}{100}\right)^n$. The value decreases by $r\%$ each period.

Finding rate or time: isolate the exponential, then take logarithms.

Question 1

Medium – Paper 2

[5 marks]

\$5 000 is invested for 3 years.

- At simple interest of 4% per annum, find the total amount at the end of 3 years.
- At compound interest of 4% per annum, find the total amount at the end of 3 years, correct to the nearest cent.
- Find the difference between the two amounts.

MISTAKE ANALYSIS

(a) Simple interest: $I = P \times r \times t = 5000 \times 0.04 \times 3 = 600$. Total = \$5600. (b) Compound: $A = 5000 \times (1.04)^3 = 5000 \times 1.124864 = \5624.32 . (c) Difference = $5624.32 - 5600 = \$24.32$. Students who apply the compound formula to part (a): $5000 \times 1.04 \times 3 = \6240 – this multiplies the factor (1.04) three times linearly instead of using simple interest $P(1 + rt)$. Simple interest grows linearly; compound interest grows exponentially. After 3 years at 4%, compound always produces more than simple, but the difference is small for short periods.

Question 2

Medium – Paper 2

[5 marks]

\$8 000 is invested at 3.5% per annum compound interest. Find the value of the investment after 5 years, giving your answer correct to 2 decimal places.

MISTAKE ANALYSIS

$A = 8000 \times (1.035)^5 = 8000 \times 1.187686 \dots = \9501.49 . *Students who calculate $8000 \times 0.035 \times 5 = \1400 (simple interest) and add: \$9400 – wrong. Compound interest applies the rate to the growing balance each year, not just the original principal. Students who write $(1+3.5)^5$ instead of $(1+0.035)^5$ use the percentage directly without converting to a decimal: $(4.5)^5$ gives a nonsensical answer. Always convert: $3.5\% = 0.035$.*

Question 3

Medium – Paper 2

[5 marks]

A car is purchased for \$25 000. It depreciates at a rate of 12% per year.

- (a) Write a formula for the value V of the car after n years.
- (b) Find the value of the car after 4 years, correct to the nearest dollar.

MISTAKE ANALYSIS

(a) $V = 25000 \times (0.88)^n$. (b) $V = 25000 \times (0.88)^4 = 25000 \times 0.59969 \dots = \14992 . Students who write $V = 25000 \times (1.12)^n$ (using $1 + 0.12$ instead of $1 - 0.12$) calculate growth, not depreciation. For depreciation, each year the car retains $100\% - 12\% = 88\%$ of its value: multiply by 0.88, not 1.12. Students who subtract 12% of the original price each year (like simple depreciation): $25000 - 4 \times 3000 = \13000 – this is straight-line depreciation. The IB model uses compound (reducing balance) depreciation.

Question 4

Hard – Paper 2

[5 marks]

\$10 000 is invested at a nominal annual rate of 6%, compounded monthly. Find the total amount after 2 years, correct to 2 decimal places.

MISTAKE ANALYSIS

Monthly rate = $6\% \div 12 = 0.5\%$. Number of compounding periods = $2 \times 12 = 24$. $A = 10000 \times \left(1 + \frac{0.06}{12}\right)^{24} = 10000 \times (1.005)^{24} = 10000 \times 1.12716\dots = \$11\,271.60$. Students who use $A = 10000 \times (1.06)^2 = \$11\,236$ apply the annual rate with annual compounding – this ignores that compounding is monthly. More frequent compounding always gives a higher final amount. Monthly compounding at 6% nominal gives an effective annual rate of $(1.005)^{12} - 1 \approx 6.168\%$, which is greater than 6%.

Question 5

Hard – Paper 2

[7 marks]

\$5 000 grows to \$6 800 in 6 years with compound interest compounded annually. Find the annual interest rate, giving your answer correct to 2 decimal places.

MISTAKE ANALYSIS

$5000 \times (1+r)^6 = 6800 \Rightarrow (1+r)^6 = 1.36 \Rightarrow 1+r = 1.36^{1/6} \Rightarrow r = 1.36^{1/6} - 1 \approx 0.05258$. Annual rate $\approx 5.26\%$. The step $(1+r)^6 = 1.36 \Rightarrow 1+r = 1.36^{1/6}$ requires taking the 6th root. Students who write $1+r = 1.36/6 = 0.2267$ divide instead of taking the root – this would give $r = -0.773$, clearly wrong. Alternatively: $(1+r)^6 = 1.36 \Rightarrow 6 \ln(1+r) = \ln 1.36 \Rightarrow r = e^{\ln(1.36)/6} - 1$ – same result via logarithms.

Question 6

Hard – Paper 2

[7 marks]

An investment earns 5% compound interest per annum. Find the minimum number of complete years for the investment to double in value.

MISTAKE ANALYSIS

$(1.05)^n > 2 \Rightarrow n > \frac{\ln 2}{\ln 1.05} \approx \frac{0.6931}{0.04879} \approx 14.21$. Minimum complete years: $n = 15$. Verify: $(1.05)^{15} \approx 2.0789 > 2 \checkmark$; $(1.05)^{14} \approx 1.9799 < 2 \checkmark$. Students who use the “Rule of 72”: $72 \div 5 = 14.4$ – a useful approximation but not an accepted IB method. Use logarithms. Students who round 14.21 down to $n = 14$ find the investment has not yet doubled. The question asks for the investment to exceed double, so round up to $n = 15$.



WORKED SOLUTIONS – SET I – FINANCIAL MATHEMATICS

M1 method · A1 accuracy · R1 reasoning

Solution – Question 1

(a) $I = 5000 \times 0.04 \times 3 = 600$; total = \$5600 M1 A1

(b) $A = 5000 \times (1.04)^3 = \5624.32 M1 A1

(c) Difference = \$24.32 A1

Solution – Question 2

$A = 8000 \times (1.035)^5 = \9501.49 M1 A1

Solution – Question 3

(a) $V = 25000 \times (0.88)^n$ M1 A1

(b) $V = 25000 \times (0.88)^4 = \14992 M1 A1

Solution – Question 4

Monthly rate = 0.5%; periods = 24; $A = 10000 \times (1.005)^{24} = \11271.60 M1 A1

Solution – Question 5

$(1 + r)^6 = 1.36 \Rightarrow 1 + r = 1.36^{1/6} \Rightarrow r = 1.36^{1/6} - 1 \approx 5.26\%$ M1 A1

Solution – Question 6

$$(1.05)^n > 2 \Rightarrow n = 15 \text{ years}$$

$$n > \frac{\ln 2}{\ln 1.05} \approx$$

14.21; round up

M1 A1 R1

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