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Differentiation

Mistake Analysis – Set III

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	39 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Optimisation method: express the objective (area, profit, distance) as a function of one variable, differentiate, set $= 0$, solve, verify it is a maximum or minimum using f''.

Combined chain + product: write out u, v, u', v' explicitly before applying the product rule. Each factor in u' or v' may itself require the chain rule.

Quotient simplification: after applying the quotient rule, look for Pythagorean identities ($\sin^2 + \cos^2 = 1$) to simplify the numerator before presenting the final answer.

Question 1 — Hard – Paper 2 — [6 marks]

A rectangular field is to be enclosed using 40 m of fencing on three sides (the fourth side is a wall). Find the dimensions that maximise the area, and state the maximum area.

MISTAKE ANALYSIS

Let width $= w$. Then length $= 40 - 2w$. Area $A = w(40 - 2w) = 40w - 2w^2$. $\frac{dA}{dw} = 40 - 4w = 0 \Rightarrow w = 10$. Length $= 20$. $\frac{d^2A}{dw^2} = -4 < 0$, confirming maximum. Maximum area $= 10 \times 20 = 200\ m^2$. Students who use all 40 m for the parallel side write length $= 40 - w$ (forgetting there are two widths). Two sides of width w use $2w$ of fencing, leaving $40 - 2w$ for the length.

Question 2 — Hard – Paper 1 — [6 marks]

Find $\dfrac{d}{dx}[(\ln x)^3]$.

MISTAKE ANALYSIS

Chain rule: outer $(\cdot)^3$*, inner* $\ln x$*.* $\dfrac{d}{dx}[(\ln x)^3] = 3(\ln x)^2 \times \dfrac{1}{x} = \dfrac{3(\ln x)^2}{x}$*. Students who write* $3(\ln x)^2$ *omit the* $\frac{1}{x}$ *factor from the inner derivative. Students who write* $3(\ln x)^2 \times \ln x = 3(\ln x)^3$ *differentiate* $\ln x$ *as* $\ln x$ *instead of* $\frac{1}{x}$*.*

Question 3 Hard – Paper 1 [6 marks]

Find $\dfrac{d}{dx}[e^x \cos(2x)]$.

MISTAKE ANALYSIS

Product rule: $u = e^x$*,* $v = \cos(2x)$*,* $u' = e^x$*,* $v' = -2\sin(2x)$ *(chain rule on* v*).* $\dfrac{d}{dx}[e^x \cos(2x)] = e^x \cos(2x) + e^x(-2\sin(2x)) = e^x(\cos(2x) - 2\sin(2x))$*. Students who write* $v' = -\sin(2x)$ *forget the chain rule factor of 2 from the inner function* $2x$*. The chain rule applies inside the product rule wherever a composite function appears.*

Question 4 Hard – Paper 1 [7 marks]

Show that $\dfrac{d}{dx}\left[\dfrac{\sin x}{1+\cos x}\right] = \dfrac{1}{1+\cos x}$.

MISTAKE ANALYSIS

Quotient rule: $u = \sin x$*,* $v = 1 + \cos x$*,* $u' = \cos x$*,* $v' = -\sin x$*.* $\dfrac{d}{dx} = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$*. Since* $\sin^2 x + \cos^2 x = 1$*: numerator* $= \cos x + 1 = (1+\cos x)$*.* $= \dfrac{1+\cos x}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$*.* ✓ *Students who stop at* $\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$ *without applying the Pythagorean identity cannot complete the simplification. Recognise* $\sin^2 x + \cos^2 x = 1$ *in the numerator.*

Question 5 Hard – Paper 1 [6 marks]

Find the equation of the tangent to $y = e^{2x}$ at $x = 0$.

MISTAKE ANALYSIS

$y(0) = e^0 = 1$. $y' = 2e^{2x}$, so $y'(0) = 2$. Tangent: $y - 1 = 2(x - 0) \Rightarrow y = 2x + 1$. Students who write $y' = e^{2x}$ (missing the chain rule factor 2) get a gradient of 1, giving the wrong tangent $y = x + 1$. Students who write $y(0) = e^0 = 0$ forget $e^0 = 1$.

Question 6 Hard – Paper 2 [8 marks]

Find all stationary points of $y = x^4 - 8x^2$ and determine their nature.

MISTAKE ANALYSIS

$y' = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x - 2)(x + 2) = 0 \Rightarrow x = 0, \pm 2$. $y'' = 12x^2 - 16$. $x = -2$: $y'' = 48 - 16 = 32 > 0$ (local minimum, $y = -16$). $x = 0$: $y'' = -16 < 0$ (local maximum, $y = 0$). $x = 2$: $y'' = 48 - 16 = 32 > 0$ (local minimum, $y = -16$). Students who find $x = 0$ and $x = \pm 2$ but report only two stationary points miss the symmetry. $4x(x^2 - 4) = 0$ has three roots, not two.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$A = w(40 - 2w)$; $\frac{dA}{dw} = 40 - 4w = 0$; $w = 10$, $l = 20$ | $A_{\max} = 200 \text{ m}^2$ | **M1 A1 R1**

Solution – Question 2

$3(\ln x)^2 \times \frac{1}{x}$ | $\dfrac{3(\ln x)^2}{x}$ | **M1 A1**

Solution – Question 3

$e^x \cos(2x) + e^x \cdot (-2\sin(2x))$ | $e^x(\cos(2x) - 2\sin(2x))$ | **M1 A1**

Solution – Question 4

Numerator $= \cos x + \cos^2 x + \sin^2 x = 1 + \cos x$ | **M1**

$\frac{1+\cos x}{(1+\cos x)^2}$ | $\dfrac{1}{1+\cos x}$ ✓ | **M1 A1**

Solution – Question 5

$y' = 2e^{2x}$; $y'(0) = 2$; $y(0) = 1$ | $y = 2x + 1$ | **M1 A1**

Solution – Question 6

$4x(x - 2)(x + 2) = 0 \Rightarrow x = 0, \pm 2$ **M1**

Min at $(\pm 2, -16)$; max at $(0, 0)$ via y'' **M1 A1 R1**
