Differentiation

Mistake Analysis – Set II

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	35 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Standard derivatives: $\frac{d}{dx}[\sin x] = \cos x$, $\frac{d}{dx}[e^x] = e^x$, $\frac{d}{dx}[\ln x] = \frac{1}{x}$.

Chain rule reminder: every composite function requires the inner derivative as a multiplying factor.

Implicit differentiation: differentiate both sides with respect to x; any y-term picks up $\frac{dy}{dx}$ via the chain rule.

Normal: gradient of normal $= -\frac{1}{f'(a)}$. Tangent and normal are perpendicular.

Question 1

Medium – Paper 1 [5 marks]

Find $\frac{d}{dx}[\sin(3x^2)]$.

MISTAKE ANALYSIS

Chain rule: outer $\sin(\cdot)$*, inner* $3x^2$*.* $\frac{d}{dx}[\sin(3x^2)] = \cos(3x^2) \times 6x = 6x\cos(3x^2)$*. Students who write* $\cos(3x^2)$ *omit the inner derivative* $6x$*. The chain rule produces a factor of* $6x$*; without it the answer is incomplete.*

Question 2

Medium – Paper 1 [4 marks]

Find $\dfrac{d}{dx}[e^{2x-1}]$.

MISTAKE ANALYSIS

$\dfrac{d}{dx}[e^{2x-1}] = e^{2x-1} \times 2 = 2e^{2x-1}$. The derivative of e^u is $e^u \cdot u'$. Students who write e^{2x-1} without the factor 2 forget that $u' = (2x-1)' = 2$. Students who write $(2x-1)e^{2x-2}$ apply the power rule to the exponential – a fundamental error. e^x differentiates to e^x, not xe^{x-1}.

Question 3 Hard – Paper 1 [5 marks]

Find $\dfrac{d}{dx}[x^3 \ln x]$.

MISTAKE ANALYSIS

Product rule: $u = x^3$, $v = \ln x$, $u' = 3x^2$, $v' = \dfrac{1}{x}$. $\dfrac{d}{dx}[x^3 \ln x] = 3x^2 \ln x + x^3 \cdot \dfrac{1}{x} = 3x^2 \ln x + x^2 = x^2(3\ln x + 1)$. Students who write $3x^2 \ln x + x^3 \ln x$ incorrectly differentiate $\ln x$ as $\ln x$ itself. $\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$, which simplifies $x^3 \cdot \dfrac{1}{x}$ to x^2.

Question 4 Hard – Paper 1 [6 marks]

A curve has equation $x^2 + y^2 = 25$. Find $\dfrac{dy}{dx}$ by implicit differentiation, and find its value at the point $(3, 4)$.

MISTAKE ANALYSIS

Differentiating both sides: $2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$. At $(3,4)$: $\dfrac{dy}{dx} = -\dfrac{3}{4}$. Students who treat y as a constant when differentiating y^2 write $\dfrac{d}{dx}[y^2] = 2y$ without the $\dfrac{dy}{dx}$ factor. Since y depends on x, the chain rule gives $\dfrac{d}{dx}[y^2] = 2y\dfrac{dy}{dx}$.

Question 5 Hard – Paper 2 [7 marks]

Find the equation of the normal to the curve $y = \sqrt{x}$ at the point where $x = 4$.

MISTAKE ANALYSIS

$y(4) = 2$. $y' = \dfrac{1}{2\sqrt{x}}$, so $y'(4) = \dfrac{1}{4}$. Normal gradient $= -\dfrac{1}{1/4} = -4$. Normal: $y - 2 = -4(x - 4) \Rightarrow y = -4x + 18$. Students who use the tangent gradient ($\frac{1}{4}$) for the normal equation find the tangent instead. The normal is perpendicular: its gradient is the negative reciprocal of the tangent gradient.

Question 6 Hard – Paper 2 [8 marks]

For the curve $y = x^3 - 3x^2 - 9x + 2$, find the intervals on which the function is increasing and decreasing.

MISTAKE ANALYSIS

$y' = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x + 1)(x - 3)$. $y' = 0$ at $x = -1$ and $x = 3$. Test signs: $y'(-2) = 3(-)(-) = 15 > 0$; $y'(0) = 3(+)(-) = -9 < 0$; $y'(4) = 3(+)(+) = 15 > 0$. Increasing: $x < -1$ and $x > 3$. Decreasing: $-1 < x < 3$. Students who give only the stationary points answer a different question. The question asks for intervals, not x-values. State the inequalities explicitly.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$\cos(3x^2) \times 6x \qquad 6x\cos(3x^2)$ **M1 A1**

Solution – Question 2

$e^{2x-1} \times 2 \qquad 2e^{2x-1}$ **M1 A1**

Solution – Question 3

$3x^2 \ln x + x^3 \cdot \frac{1}{x} = \quad x^2(3\ln x + 1)$ **M1 A1**
$3x^2 \ln x + x^2$

Solution – Question 4

$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \quad -\frac{3}{4}$ **M1 A1**
$\frac{dy}{dx} \quad = \quad -\frac{x}{y}$; at
$(3, 4)$

Solution – Question 5

$y'(4) \qquad = \quad y = -4x + 18$ **M1 A1**
$\frac{1}{4}$; normal
gradient
$= \ -4; \ y \ - \ 2 \ =$
$-4(x - 4)$

Solution – Question 6

$3(x+1)(x-3) = 0 \Rightarrow x = -1, 3$; sign test	**M1**
Increasing $x < -1$ and $x > 3$; decreasing $-1 < x < 3$	**A1 R1**