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Differentiation

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches SL
Topic	Topic 5 – Calculus
Level	Medium → Hard (Paper 1 and Paper 2 style)
Questions	6
Marks	34 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Power rule: $\frac{d}{dx}[x^n] = nx^{n-1}$.

Chain rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$. The derivative of the outer function, times the derivative of the inner.

Product rule: $\frac{d}{dx}[uv] = u'v + uv'$.

Quotient rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$.

Tangent line: gradient $= f'(a)$; equation $y - f(a) = f'(a)(x - a)$.

Question 1

Medium – Paper 1 [4 marks]

Find $f'(x)$ given $f(x) = 3x^4 - 2x^2 + 5x - 1$.

MISTAKE ANALYSIS

$f'(x) = 12x^3 - 4x + 5$. Differentiate each term: $3x^4 \to 12x^3$, $-2x^2 \to -4x$, $5x \to 5$, $-1 \to 0$. Students who write $f'(x) = 12x^3 - 4x + 5 - 1$ keep the constant -1. Constants differentiate to zero, not to themselves.

Question 2

Medium – Paper 1 [4 marks]

Find $f'(x)$ given $f(x) = (3x+2)^5$.

MISTAKE ANALYSIS

Chain rule: $f'(x) = 5(3x+2)^4 \times 3 = 15(3x+2)^4$. *Students who write* $5(3x+2)^4$ *forget to multiply by the derivative of the inner function* $(3x+2)$, *which is 3. The chain rule requires that extra factor every time.*

Question 3

Hard – Paper 1 [6 marks]

Find $f'(x)$ given $f(x) = x^2 \sin x$.

MISTAKE ANALYSIS

Product rule with $u = x^2$, $v = \sin x$: $u' = 2x$, $v' = \cos x$. $f'(x) = 2x \sin x + x^2 \cos x$. *Students who write* $f'(x) = 2x \cos x$ *differentiate each factor separately and multiply – this is not the product rule. The rule is* $u'v + uv'$, *giving two terms, not one.*

Question 4

Hard – Paper 1 [6 marks]

Find $f'(x)$ given $f(x) = \dfrac{2x+1}{x^2+3}$.

MISTAKE ANALYSIS

Quotient rule: $u = 2x+1$, $v = x^2+3$, $u' = 2$, $v' = 2x$. $f'(x) = \dfrac{2(x^2+3) - (2x+1)(2x)}{(x^2+3)^2} = \dfrac{2x^2+6-4x^2-2x}{(x^2+3)^2} = \dfrac{-2x^2-2x+6}{(x^2+3)^2}$. *Students who write* $\frac{u'}{v'} = \frac{2}{2x} = \frac{1}{x}$ *divide derivatives instead of applying the quotient rule. The quotient rule is not* $\frac{u'}{v'}$.

Question 5 Hard – Paper 2 [7 marks]

Find the equation of the tangent to $y = x^3 - 4x$ at the point where $x = 2$.

MISTAKE ANALYSIS

$y(2) = 8-8 = 0$. Point: $(2, 0)$. $y' = 3x^2-4$, so $y'(2) = 12-4 = 8$. Tangent: $y-0 = 8(x-2) \Rightarrow y = 8x-16$. Students who substitute $x = 2$ into the original function to find the gradient ($y(2) = 0$) confuse the function value with the derivative value. The gradient comes from $f'(2)$, not $f(2)$.

Question 6 Hard – Paper 2 [7 marks]

Find the x-coordinates of the stationary points of $y = 2x^3 - 9x^2 + 12x - 1$ and determine their nature.

MISTAKE ANALYSIS

$y' = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2) = 0 \Rightarrow x = 1$ or $x = 2$. $y'' = 12x - 18$. At $x = 1$: $y'' = -6 < 0$ (local maximum, $y = 4$). At $x = 2$: $y'' = 6 > 0$ (local minimum, $y = 3$). Students who give only the x-values without using y'' to determine the nature lose the R1 mark. State the sign of y'' and its conclusion explicitly.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

Power rule each term	$12x^3 - 4x + 5$	**M1 A1**

Solution – Question 2

$5(3x+2)^4 \times 3$	$15(3x+2)^4$	**M1 A1**

Solution – Question 3

$u = x^2,\ v = \sin x;\ u'v + uv'$	$2x\sin x + x^2\cos x$	**M1 A1**

Solution – Question 4

$\frac{2(x^2+3)-(2x+1)(2x)}{(x^2+3)^2}$	$\dfrac{-2x^2-2x+6}{(x^2+3)^2}$	**M1 A1**

Solution – Question 5

$y(2) = 0;\ y'(2) = 8;\ y - 0 = 8(x-2)$	$y = 8x - 16$	**M1 A1**

Solution – Question 6

$6(x-1)(x-2) = 0 \Rightarrow x = 1, 2$		**M1 A1**
$y''(1) = -6 < 0$ (max); $y''(2) = 6 > 0$ (min)	Max at $(1,4)$, min at $(2,3)$	**M1 A1 R1**