

Arithmetic & Geometric Sequences

Recognition Training – Set I

Course	IB Mathematics: Analysis & Approaches HL
Topic	Topic 1 – Number & Algebra
Level	Easy → Medium
Questions	6
Total marks	30
Instructions	Show all working. M1 = method mark (correct process). A1 = accuracy mark (correct result). R1 = reasoning mark. Do not use a calculator unless stated.

BEFORE YOU BEGIN

An **arithmetic sequence** has first term u_1 and common difference d . The general term is $u_n = u_1 + (n - 1)d$ and the sum of the first n terms is $S_n = \frac{n}{2}(2u_1 + (n - 1)d) = \frac{n}{2}(u_1 + u_n)$.

A **geometric sequence** has first term u_1 and common ratio r . The general term is $u_n = u_1 r^{n-1}$ and the sum of the first n terms is $S_n = \frac{u_1(r^n - 1)}{r - 1}$ for $r \neq 1$.

Write the formula, substitute values, then solve. Never substitute before identifying the correct formula.

Question 1

Easy

[4 marks]

An arithmetic sequence has first term 3 and common difference 4.

- Write down the 10th term.
- Find the sum of the first 10 terms.

MISTAKE ANALYSIS

$u_{10} = u_1 + (10 - 1)d = 3 + 9 \times 4 = 39$. $S_{10} = \frac{10}{2}(u_1 + u_{10}) = 5(3 + 39) = 210$. *The error: using $n = 10$ instead of $(n - 1) = 9$ in the general term formula, writing $u_{10} = 3 + 10 \times 4 = 43$. The formula is $u_1 + (n - 1)d$, not $u_1 + nd$. Count the differences: between term 1 and term n there are $n - 1$ steps, not n .*

Question 2

Easy

[4 marks]

A geometric sequence has first term 2 and common ratio 3. Find the 6th term and the sum of the first 6 terms.

MISTAKE ANALYSIS

$u_6 = 2 \times 3^5 = 2 \times 243 = 486$. $S_6 = \frac{2(3^6 - 1)}{3 - 1} = \frac{2 \times 728}{2} = 728$. *The error: writing $u_6 = 2 \times 3^6 = 1458$ – using r^n instead of r^{n-1} . In a geometric sequence, the first term has $r^0 = 1$ (no ratio applied), so the n th term has r^{n-1} . Verify: $u_1 = 2 \times 3^0 = 2 \checkmark$.*

Question 3

Easy–Medium

[5 marks]

The 4th term of an arithmetic sequence is 13 and the 9th term is 33. Find the first term and common difference, and hence find the sum of the first 20 terms.

MISTAKE ANALYSIS

Set up two equations: $u_1 + 3d = 13$ and $u_1 + 8d = 33$. Subtract: $5d = 20$, so $d = 4$ and $u_1 = 1$. $S_{20} = \frac{20}{2}(2 \times 1 + 19 \times 4) = 10 \times 78 = 780$. Students who write $u_4 = 4u_1$ and $u_9 = 9u_1$ are confusing arithmetic with geometric sequences. In an arithmetic sequence, $u_n = u_1 + (n - 1)d$ – the index multiplies d , not u_1 .

Question 4

Medium

[5 marks]

The sum of the first n terms of an arithmetic sequence is given by $S_n = 2n^2 + 3n$. Find the first term, the common difference, and the 15th term.

MISTAKE ANALYSIS

$u_1 = S_1 = 2 + 3 = 5$. $u_n = S_n - S_{n-1} = (2n^2 + 3n) - (2(n-1)^2 + 3(n-1)) = 4n + 1$. So $d = u_2 - u_1 = (4 \times 2 + 1) - (4 \times 1 + 1) = 4$. Check: $u_1 = 4(1) + 1 = 5 \checkmark$. $u_{15} = 4(15) + 1 = 61$. Students who differentiate S_n with respect to n to find u_n write $u_n = 4n + 3$ – this is the derivative, not the discrete difference. The correct approach is $u_n = S_n - S_{n-1}$, not $\frac{dS_n}{dn}$.

Question 5

Medium

[6 marks]

The 3rd term of a geometric sequence is $\frac{4}{3}$ and the 6th term is $\frac{32}{81}$. Find the first term and common ratio, and hence find the sum of the first 8 terms.

MISTAKE ANALYSIS

$u_3 = u_1 r^2 = \frac{4}{3}$ and $u_6 = u_1 r^5 = \frac{32}{81}$. Divide: $r^3 = \frac{32/81}{4/3} = \frac{32}{81} \times \frac{3}{4} = \frac{8}{27}$, so $r = \frac{2}{3}$. $u_1 = \frac{4/3}{(2/3)^2} = \frac{4/3}{4/9} = 3$. $S_8 = \frac{3(1 - (2/3)^8)}{1 - 2/3} = 9 \left(1 - \frac{256}{6561}\right) = 9 \times \frac{6305}{6561} = \frac{6305}{729}$. The error: dividing the equations in the wrong order, writing $r^3 = \frac{4/3}{32/81}$ and getting $r = \frac{3}{2}$. Always divide the higher-indexed term by the lower-indexed term to get a positive power of r .

Question 6

Medium

[6 marks]

The first three terms of a sequence are $k + 2$, $2k - 1$, $4k - 3$, where k is a constant.

- (a) Given that the sequence is arithmetic, find the value of k and the common difference.
- (b) Given instead that the sequence is geometric, find the possible values of k and the corresponding common ratios.

MISTAKE ANALYSIS

Arithmetic: equal differences: $(2k - 1) - (k + 2) = (4k - 3) - (2k - 1)$. $k - 3 = 2k - 2$, so $k = -1$, giving terms 1, -3 , -7 and $d = -4$.

Geometric: equal ratios: $\frac{2k - 1}{k + 2} = \frac{4k - 3}{2k - 1}$. Cross-multiply: $(2k - 1)^2 = (4k - 3)(k + 2)$. $4k^2 - 4k + 1 = 4k^2 + 5k - 6$, so $9k = 7$, giving $k = \frac{7}{9}$. Terms: $\frac{25}{9}$, $\frac{5}{9}$, $\frac{1}{9}$, common ratio $\frac{1}{5}$.

The error in part (b): students write the geometric condition as $u_2 - u_1 = u_3 - u_2$ (the arithmetic condition) rather than $\frac{u_2}{u_1} = \frac{u_3}{u_2}$. Equal ratios, not equal differences.



WORKED SOLUTIONS – SET I – ARITHMETIC & GEOMETRIC SEQUENCES

M1 = method mark. A1 = accuracy mark. R1 = reasoning mark.

Solution – Question 1

$$(a) \quad u_{10} = u_1 + 3 + 9 \times 4 = 3 + 36 = 39 \quad \text{M1}$$

$$(b) \quad S_{10} = 5 \times (3 + 39) = 5 \times 42 = 210 \quad \text{M1}$$
$$\frac{10}{2}(u_1 + u_{10})$$

Answers: $u_{10} = 39, \quad S_{10} = 210$

Solution – Question 2

$$u_6 = u_1 \cdot r^5 \quad 2 \times 3^5 = 2 \times 243 = 486 \quad \text{M1}$$

$$S_6 = \frac{u_1(r^6 - 1)}{r - 1} \quad \frac{2(729 - 1)}{2} = 728 \quad \text{M1}$$

Answers: $u_6 = 486, \quad S_6 = 728$

Solution – Question 3

Write equations $u_1 + 3d = 13$ and $u_1 + 8d = 33$ M1

Subtract $5d = 20 \Rightarrow d = 4, \quad u_1 = 1$ A1

$$S_{20} = \frac{20}{2}(2u_1 + 19d) = 10(2 + 76) = 10 \times 78 = 780 \quad \text{M1}$$

Final answer: $u_1 = 1, \quad d = 4, \quad S_{20} = 780$

Solution – Question 4

$$u_1 = S_1 \quad 2(1)^2 + 3(1) = 5 \quad \text{M1}$$

$$u_n = S_n - S_{n-1} \quad (2n^2 + 3n) - (2n^2 - n - 1 + 3n - 3) = 4n + 1 \quad \text{M1}$$

$$d = u_2 - u_1 \quad (4 \times 2 + 1) - (4 \times 1 + 1) = 4 \quad \text{A1}$$

$$u_{15} = 4(15) + 1 \quad 61 \quad \text{A1}$$

Final answer: $u_1 = 5, \quad d = 4, \quad u_{15} = 61$

Solution – Question 5

Write equations $u_1 r^2 = \frac{4}{3}$ and $u_1 r^5 = \frac{32}{81}$ M1

Divide (higher by lower) $r^3 = \frac{32/81}{4/3} = \frac{8}{27} \Rightarrow r = \frac{2}{3}$ A1

Find u_1 $u_1 = \frac{4/3}{4/9} = 3$ A1

$S_8 = \frac{3(1-(2/3)^8)}{1-2/3} = \frac{3(1-256/6561)}{1/3} = 9 \times \frac{6305}{6561} = \frac{6305}{729}$ M1

Final answer: $u_1 = 3, r = \frac{2}{3}, S_8 = \frac{6305}{729}$

Solution – Question 6**Part (a) – Arithmetic:**

Equal differences $(2k - 1) - (k + 2) = (4k - 3) - (2k - 1)$ M1

Simplify $k - 3 = 2k - 2 \Rightarrow k = -1$ A1

Common difference $d = (2(-1) - 1) - (-1 + 2) = -3 - 1 = -4$ A1

Part (b) – Geometric:

Equal ratios $(2k - 1)^2 = (k + 2)(4k - 3)$ M1

Expand and simplify $4k^2 - 4k + 1 = 4k^2 + 5k - 6 \Rightarrow 9k = 7 \Rightarrow k = \frac{7}{9}$ A1

Common ratio $r = \frac{2k - 1}{k + 2} = \frac{5/9}{25/9} = \frac{1}{5}$ A1

Final answer: (a) $k = -1, d = -4$ (b) $k = \frac{7}{9}, r = \frac{1}{5}$
