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Differentiation

Mistake Analysis – Set I

Course	IB Mathematics: Analysis & Approaches HL
Topic	Topic 5 – Calculus: Differentiation
Level	Easy → Medium
Questions	6
Marks	32 total. **M1** method · **A1** accuracy · **R1** reasoning.

BEFORE YOU BEGIN

Power rule: $\frac{d}{dx}[x^n] = nx^{n-1}$ for all $n \in \mathbb{R}$.

Product rule: $\frac{d}{dx}[uv] = u'v + uv'$.

Quotient rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v-uv'}{v^2}$.

Chain rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$.

Standard derivatives: $\frac{d}{dx}[e^x] = e^x$; $\frac{d}{dx}[\ln x] = \frac{1}{x}$; $\frac{d}{dx}[\sin x] = \cos x$; $\frac{d}{dx}[\cos x] = -\sin x$.

Question 1 — Easy — [4 marks]

Differentiate $f(x) = 3x^4 - 2x^3 + 5x - 7$ and find $f'(2)$.

MISTAKE ANALYSIS

$f'(x) = 12x^3 - 6x^2 + 5$. $f'(2) = 12(8) - 6(4) + 5 = 96 - 24 + 5 = 77$. *Students who forget to differentiate the constant* (-7) *and instead write* $f'(x) = 12x^3 - 6x^2 + 5 - 7$ *make a systematic error. Constants vanish under differentiation. Also: the derivative of* $5x$ *is* 5*, not* $5x$ *or* 1.

Question 2 — Easy — [4 marks]

Differentiate $f(x) = \sqrt{x} + \dfrac{3}{x^2}$. Simplify your answer.

MISTAKE ANALYSIS

Rewrite: $f(x) = x^{1/2} + 3x^{-2}$. $f'(x) = \frac{1}{2}x^{-1/2} - 6x^{-3} = \dfrac{1}{2\sqrt{x}} - \dfrac{6}{x^3}$. *Students who differentiate* $\sqrt{x}$ *as* $\frac{1}{\sqrt{x}}$ *(omitting the factor* $\frac{1}{2}$*) get* $x^{1/2} \to x^{-1/2}$ *without the coefficient. The power rule says* $\frac{d}{dx}[x^n] = nx^{n-1}$*: the exponent* $\frac{1}{2}$ *multiplies down. Also:* $3x^{-2}$ *differentiates to* $-6x^{-3}$ *(multiply by* -2*, reduce exponent by 1), not* $-3x^{-3}$.

Question 3 Medium [5 marks]

Find $\dfrac{dy}{dx}$ where $y = x^2e^x$. Hence find the x-coordinates where the gradient is zero.

MISTAKE ANALYSIS

Product rule: $u = x^2$, $v = e^x$; $u' = 2x$, $v' = e^x$. $\frac{dy}{dx} = 2xe^x + x^2e^x = e^x(2x + x^2) = xe^x(x + 2)$. *Set to zero:* $xe^x(x + 2) = 0$. *Since* $e^x > 0$ *always:* $x = 0$ *or* $x = -2$. *The most common error: treating* e^x *as a constant and writing* $\frac{d}{dx}[x^2e^x] = 2xe^x$. e^x *is a function of* x*; the product rule is mandatory. Also:* $e^x = 0$ *has no solution. State this when solving – it earns the R1 mark.*

Question 4 Medium [5 marks]

Differentiate $y = \dfrac{x^2 + 1}{x - 1}$ using the quotient rule. Find $\dfrac{dy}{dx}$ at $x = 2$.

MISTAKE ANALYSIS

$u = x^2 + 1$, $v = x - 1$; $u' = 2x$, $v' = 1$. $\frac{dy}{dx} = \frac{2x(x-1)-(x^2+1)}{(x-1)^2} = \frac{2x^2-2x-x^2-1}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$. *At* $x = 2$*:* $\frac{4-4-1}{1} = -1$. *The most common error: writing the quotient rule as* $\frac{u'v'-uv}{v^2}$ *or* $\frac{uv'-u'v}{v^2}$ *(wrong order). The correct form is* $\frac{u'v-uv'}{v^2}$*: numerator derivative minus denominator derivative. A mnemonic: "low-d-high minus high-d-low over low-squared."*

Question 5 Medium [6 marks]

Find $\frac{dy}{dx}$ for each of the following using the chain rule:

(a) $y = (3x^2 + 1)^5$

(b) $y = \sin(2x + 1)$

(c) $y = \ln(x^2 + 3)$

MISTAKE ANALYSIS

(a) $\frac{dy}{dx} = 5(3x^2+1)^4 \cdot 6x = 30x(3x^2+1)^4$. *(b)* $\frac{dy}{dx} = \cos(2x+1) \cdot 2 = 2\cos(2x+1)$. *(c)* $\frac{dy}{dx} = \frac{1}{x^2+3} \cdot 2x = \frac{2x}{x^2+3}$. *The chain rule requires multiplying by the derivative of the inner function. Students who write (a) as* $5(3x^2+1)^4$ *(forgetting to differentiate* $3x^2+1$*) or (c) as* $\frac{1}{x^2+3}$ *(forgetting the factor* $2x$*) omit the outer-times-inner structure. For (c):* $\frac{d}{dx}[\ln u] = \frac{u'}{u}$*, not* $\frac{1}{u}$.

Question 6 Medium [8 marks]

(a) Find $\frac{dy}{dx}$ implicitly from $x^2 + y^2 = 25$. Find the gradient at $(3, 4)$.

(b) The curve $y = x^3 - 3x$ has a tangent at $x = 2$. Find the equation of the tangent and the equation of the normal at this point.

MISTAKE ANALYSIS

(a) Differentiate both sides with respect to x*:* $2x + 2y\frac{dy}{dx} = 0$*, so* $\frac{dy}{dx} = -\frac{x}{y}$*. At* $(3, 4)$*: gradient* $= -\frac{3}{4}$*. Check:* $(3, 4)$ *lies on the circle since* $9 + 16 = 25$*. ✓ (b)* $y(2) = 8 - 6 = 2$*.* $y' = 3x^2 - 3$*;* $y'(2) = 12 - 3 = 9$*. Tangent:* $y - 2 = 9(x - 2) \Rightarrow y = 9x - 16$*. Normal slope:* $-\frac{1}{9}$*. Normal:* $y - 2 = -\frac{1}{9}(x - 2) \Rightarrow x + 9y = 20$*. For implicit differentiation, students who write* $2x + 2y = 0$ *(forgetting* $\frac{dy}{dx}$ *on the* y^2 *term) treat* y *as a constant. Always apply the chain rule when differentiating* y *terms:* $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$.

M1 method · **A1** accuracy · **R1** reasoning

Solution – Question 1

$f'(x) = 12x^3 - 6x^2 + 5$		**M1**
$f'(2) = 96 - 24 + 5$	77	**A1**

Solution – Question 2

$f(x) = x^{1/2} + 3x^{-2}$; power rule applied term by term	$\frac{1}{2\sqrt{x}} - \frac{6}{x^3}$	**M1**

Solution – Question 3

Product rule: $u = x^2$, $v = e^x$	$xe^x(x+2)$	**M1**
$xe^x(x+2) = 0$; $e^x > 0$ always	$x = 0$ or $x = -2$	**R1**

Solution – Question 4

Quotient rule: $u'v - uv' = (2x)(x-1) - (x^2+1)(1)$	$\frac{x^2 - 2x - 1}{(x-1)^2}$	**M1**
At $x = 2$: numerator $= -1$, denominator $= 1$	-1	**A1**

Solution – Question 5

(a) Chain: $5(3x^2+1)^4 \cdot 6x$ $30x(3x^2+1)^4$ **M1**

(b) Chain: $\cos(2x+1) \cdot 2$ $2\cos(2x+1)$ **A1**

(c) Chain: $\frac{1}{x^2+3} \cdot 2x$ $\dfrac{2x}{x^2+3}$ **A1**

Solution – Question 6

(a) $2x+2y\frac{dy}{dx}=0$ $\dfrac{dy}{dx}=-\dfrac{x}{y}$; at $(3,4):\ -\dfrac{3}{4}$ **M1**

(b) $y(2)=2$; $y'(2)=9$; tangent: $y=9x-16$ **M1**

Normal slope $=-1/9$; normal: $x+9y=20$ **A1**
