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Integration by Parts

Recognition Training · Set II

Course	IB Mathematics: Analysis & Approaches HL
Topic	Topic 5 — Calculus
Level	Medium → Hard
Questions	6
Total marks	30
Instructions	Show all working. **M1** = method mark (correct process). **A1** = accuracy mark (correct result). **R1** = reasoning mark. Do not use a calculator unless stated.

BEFORE YOU BEGIN

Integration by parts applies the formula $\displaystyle\int u\,dv = uv - \int v\,du$. The entire technique depends on one decision made before any calculation begins: which factor becomes u and which becomes dv. Use the **LIATE** order to guide that decision — Logarithm, Inverse trig, Algebraic, Trigonometric, Exponential — choosing u as whichever type appears first in the list. The signal that your assignment is wrong: the new integral $\int v\,du$ is *more* complex than the original. Stop. Reassign. Restart.

Question 1 — Medium — [5 marks]

Find the following integral.

$$\int x\,e^x\,dx$$

MISTAKE ANALYSIS

The classic entry-level integration by parts question. By LIATE, $u = x$ (Algebraic) and $dv = e^x\,dx$ (Exponential). Students who assign $u = e^x$ produce a new integral $\int e^x \cdot \frac{x^2}{2}\,dx$ — more complex, not simpler. The signal is immediate: if the new integral is harder, the assignment is wrong. Stop and reassign.

Question 2 Medium [5 marks]

Find the following integral.

$$\int x \cos x \, dx$$

MISTAKE ANALYSIS

By LIATE, $u = x$ (Algebraic) and $dv = \cos x \, dx$ (Trigonometric). The most common error is a sign error when differentiating or integrating the trigonometric factor: students write $\int \sin x \, dx = \cos x + C$ instead of $-\cos x + C$. Check the sign of every trigonometric integral explicitly before writing the final answer.

Question 3 Medium [5 marks]

Find the following integral.

$$\int \ln x \, dx$$

MISTAKE ANALYSIS

There is only one factor visible, yet integration by parts still applies. Write $\ln x = \ln x \cdot 1$ and assign $u = \ln x$, $dv = 1 \, dx$. By LIATE, Logarithm takes priority. Students who cannot identify dv when there is no obvious second factor attempt substitution or leave the question. The rewrite $\int \ln x \cdot 1 \, dx$ is the key step.

Question 4 Hard [5 marks]

Find the following integral.

$$\int x^2 e^x \, dx$$

MISTAKE ANALYSIS

This requires integration by parts twice. First application: $u = x^2$, $dv = e^x \, dx$ gives $x^2 e^x - \int 2x \, e^x \, dx$. The second integral $\int 2x \, e^x \, dx$ requires a second application. Students apply by parts once, see another integral *remaining, and conclude they have made an error. They have not. Continue applying until the integral reduces to zero degree in the algebraic factor.*

Question 5 Hard [5 marks]

Find the following integral.

$$\int e^x \sin x \, dx$$

MISTAKE ANALYSIS

This is the cyclic case. After two applications of integration by parts, the original integral reappears on the right-hand side. Students who reach $\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$ conclude they have gone in a circle and start again. Do not restart. Collect the integral: add $\int e^x \sin x \, dx$ to both sides and divide by 2. This is the intended method.

Question 6 Hard [5 marks]

Evaluate the following definite integral.

$$\int_0^1 x \, e^x \, dx$$

MISTAKE ANALYSIS

This is Q1 converted to a definite integral. The technique is identical: $u = x$, $dv = e^x \, dx$. The error occurs at the evaluation stage: students substitute the limits into uv only, forgetting to also evaluate $\left[-\int v \, du\right]_0^1$. Every term in the expression must be evaluated between the limits. Write $\left[xe^x - e^x\right]_0^1$ explicitly before substituting.

M1 = method mark (correct process). **A1** = accuracy mark (correct result). **R1** = reasoning mark.

Solution — Question 1

Assign	$u = x, \quad dv = e^x\,dx \;\Rightarrow\; du = dx, \quad v = e^x$	**M1**
Apply formula	$\int x\,e^x\,dx = xe^x - \int e^x\,dx$	**M1**
Integrate	$xe^x - e^x + C$	**A1**
Factorise	$e^x(x-1) + C$	**A1**

Final answer: $e^x(x-1) + C$

Solution — Question 2

Assign	$u = x, \quad dv = \cos x\,dx \;\Rightarrow\; du = dx, \quad v = \sin x$	**M1**
Apply formula	$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$	**M1**
Integrate	$x\sin x - (-\cos x) + C$	**A1**
Simplify	$x\sin x + \cos x + C$	**A1**

Final answer: $x\sin x + \cos x + C$

Solution — Question 3

Rewrite	$\int \ln x\,dx = \int \ln x \cdot 1\,dx$	**R1**
Assign	$u = \ln x, \quad dv = dx \;\Rightarrow\; du = \frac{1}{x}\,dx, \quad v = x$	**M1**
Apply formula	$x\ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int 1\,dx$	**M1**
Integrate	$x\ln x - x + C$	**A1**

Final answer: $x\ln x - x + C$

Solution — Question 4

First application	$u = x^2,\ dv = e^x dx \ \Rightarrow\ du = 2x\,dx,\ v = e^x$	**M1**
Result	$x^2e^x - \int 2x\,e^x\,dx$	**M1**
Second application	$u = 2x,\ dv = e^x dx \ \Rightarrow\ du = 2\,dx,\ v = e^x$	**M1**
Second result	$x^2e^x - \left(2xe^x - \int 2e^x\,dx\right)$	**A1**
Integrate & simplify	$x^2e^x - 2xe^x + 2e^x + C = e^x(x^2 - 2x + 2) + C$	**A1**

Final answer: $e^x(x^2 - 2x + 2) + C$

Solution — Question 5

First application	$u = \sin x,\ dv = e^x dx \ \Rightarrow\ du = \cos x\,dx,\ v = e^x$	**M1**
First result	$e^x \sin x - \int e^x \cos x\,dx$	**M1**
Second application	$u = \cos x,\ dv = e^x dx \ \Rightarrow\ du = -\sin x\,dx,\ v = e^x$	**M1**
Second result	$e^x \sin x - \left(e^x \cos x + \int e^x \sin x\,dx\right)$	**A1**
Collect integral	$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$	**A1**

Dividing both sides by 2:

$$\int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} + C$$

Final answer: $\dfrac{e^x(\sin x - \cos x)}{2} + C$

Solution — Question 6

Assign	$u = x, \quad dv = e^x dx \ \Rightarrow\ du = dx, \quad v = e^x$	**M1**
Apply formula	$[xe^x]_0^1 - [e^x]_0^1$	**M1**
Evaluate uv	$[xe^x - e^x]_0^1 = [e^x(x-1)]_0^1$	**M1**
Substitute limits	$e^1(1-1) - e^0(0-1) = 0 - (-1)$	**A1**
Evaluate	1	**A1**

Final answer: 1
